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Performance Test of Fluid-Structure Interaction Analysis of a Sloshing Problem

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Global linear equations are not symmetric in the typical finite element formulation for a sloshing fluid-structure interaction problem. There are three approaches to solve the non-symmetric equations: (a) directly solving by a non-symmetric matrix solver; (b) transforming the non-symmetric equations into symmetric ones explicitly and then solving by a symmetric matrix solver; (c) implementing the transformation procedure into a symmetric matrix solver implicitly. In the present paper, the performance comparison of these three methods is given for a sloshing problem.
Key Words: *Fluid-Structure Interaction; Finite Element; Non-symmetric solver*

1. INTRODUCTION

In the typical finite element formulation for a sloshing fluid-structure interaction problem with freedoms of displacements and pressures and wave heights, the global linear equations are not symmetric. The non-symmetric equations can be solved directly by a non-symmetric matrix solver or be solved by a symmetric matrix solver after being transformed into symmetric equations. The transformation can be taken explicitly before solving or be implemented into the matrix solver implicitly. So there are three approaches: (a) directly solving by a non-symmetric matrix solver; (b) transforming the non-symmetric equations into symmetric ones explicitly and then solving by a symmetric matrix solver; (c) implementing the transformation procedure into a symmetric matrix solver implicitly. In this paper, a sloshing problem will be solved by the three methods and the performance comparison will be given for eigen analysis between the method (b) and (c) and for dynamic response analysis between the method (a) and (b).

2. FORMULATIONS

Using the finite element analysis interpolations [1] [2], the structural dynamics equations can be discretized as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_o + \mathbf{f}_p \tag{1}$$

where $\mathbf{u}$ represents the node displacement vector and the displacement in the structure domain is interpolated as:

$$u = \mathbf{N}\mathbf{u} \tag{2}$$

where $\mathbf{N}$ is the interpolation function vector. $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the dumping matrix and $\mathbf{K}$ is the stiffness matrix. $\mathbf{f}_o$ is the external node force vector except the fluid pressure and $\mathbf{f}_p$ is the node force vector due to the fluid pressure:

$$\mathbf{f}_p = \int_{S_I} \mathbf{N}^T n \mathbf{N}_p \mathbf{p} ds = \int_{S_I} \mathbf{N}^T n \mathbf{N}_p ds \mathbf{p} = \mathbf{Q}\mathbf{p} \tag{3}$$

where $\mathbf{p}$ is the vector that contains nodal values of fluid pressure and $\mathbf{N}_p$ is the interpolation function vector of fluid pressure.

The fluid domain dominated by the Navier-Stocks equations and the continuity equation can be simplified as a pressure field, assuming that the fluid is incompressible and ignoring the effects of nonlinear convective terms as well as the viscous terms.

$$\nabla^2 p - \frac{1}{c^2}\ddot{p} = 0 \tag{4}$$

where p denotes the pressure and c is the fluid sound speed.

On the fluid-structure interaction interfaces S_I, the acceleration normal to the interface surface is continuous and the equilibrium equation of Newton's second law for a fluid particle becomes:

$$\frac{\partial p}{\partial n} = \rho n \cdot \ddot{u} \tag{5}$$

where ρ is the fluid density, n is the normal direction of the interface surface from the solid domain to the fluid domain and u denotes the displacement.

On the free surface, the pressure can be approximately expressed for small-amplitude gravity waves as:

$$p = \overline{p} + \rho g \eta \tag{6}$$

where g is the gravity and η denotes the wave elevation with respect to the equilibrium free surface and $\overline{p}$ denotes the prescribed part of the pressure, for example, atmospheric pressure. For a fluid particle on the free surface, the following condition is satisfied.

$$\frac{\partial p}{\partial \eta} = \rho \ddot{\eta} \tag{7}$$

Applying the variational principle for equation (4) and (6), and considering equations (5) and (7), we can have

$$\mathbf{S}\ddot{\mathbf{p}} + \mathbf{H}\mathbf{p} + \rho \mathbf{Q}^T \ddot{\mathbf{u}} + \rho \mathbf{T}_{\eta p}{}^T \ddot{\boldsymbol{\eta}} = 0 \tag{8}$$

$$\mathbf{K}_\eta \boldsymbol{\eta} - \mathbf{T}_{\eta p}\mathbf{p} = 0 \tag{9}$$

where

$$\mathbf{S} = \int_V \frac{1}{c^2} \mathbf{N}_p{}^T \mathbf{N}_p dV \tag{10}$$

$$\mathbf{H} = \int_V \left(\frac{\partial N_p}{\partial x}^T \frac{\partial N_p}{\partial x} + \frac{\partial N_p}{\partial y}^T \frac{\partial N_p}{\partial y} + \frac{\partial N_p}{\partial z}^T \frac{\partial N_p}{\partial z}\right) dV \tag{11}$$

$$\mathbf{T}_{\eta p} = \int_S \mathbf{N}_p{}^T \mathbf{N}_p dS \tag{12}$$

$$\mathbf{K}_\eta = \rho g \int_S \mathbf{N}_p{}^T \mathbf{N}_p dS \tag{13}$$

Rewriting equations (1), (8) and (9), yields the global linear equations to be solved.

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \rho \mathbf{Q}^T & \rho \mathbf{T}_{\eta p}{}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\eta}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{Q} \\ \mathbf{0} & \mathbf{K}_\eta & -\mathbf{T}_{\eta p} \\ \mathbf{0} & \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\eta} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_o \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{14}$$

in which the damping is no included here and $\mathbf{S}$ is set approximately to 0.

Eliminating the pressure yields the following symmetric equations.

$$\begin{bmatrix} \mathbf{M} + \mathbf{M}_f & \mathbf{Q}_\eta \\ \mathbf{Q}_\eta{}^T & \mathbf{M}_\eta \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\eta}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_\eta \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\eta} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_o \\ \mathbf{0} \end{Bmatrix} \tag{15}$$

where

$$\mathbf{M}_f = \rho \mathbf{Q}\mathbf{H}^{-1}\mathbf{Q}^T \tag{16}$$

$$\mathbf{Q}_\eta = \rho \mathbf{Q}\mathbf{H}^{-1}\mathbf{T}_{\eta p}{}^T \tag{17}$$

$$\mathbf{M}_\eta = \rho \mathbf{T}_{\eta p}\mathbf{H}^{-1}\mathbf{T}_{\eta p}{}^T \tag{18}$$

In the method (a) the equation (14) is solved, while in the method (b) the equation (15) is solved. In the method (c) the calculation of equations (16), (17) and (18) in implemented into a symmetric matrix solver and matrices M_f, $\mathbf{Q}_\eta$ and M_η are not saved explicitly.

3. PERFORMANCE TEST

Eigen-modes and Seismic response of a cylindrical tank partly filled with water is calculated with the FEM model shown as Fig. 1, whose dimensions are: the diameter D=1760mm, the height H= 289.7mm, the wall thickness t=25mm. This tank has Young's modulus E=1.62×10^5MPa, Poisson ratio ν =0.3 and the density ρ =7.551 × 10^{-6}kg/mm^3.Water is filled to a height of 236.2mm. The density of water is given as 1.019×10^{-6}kg/mm^3 . There are 8100 fluid elements and 1785 shell elements in the FEM model.

3.1 Eigen Analysis

5 eigen modes are extracted by the subspace method. One of the modes is shown in Fig.2. The CPU time of method (b) is 5.9 times more than that of the method (c).

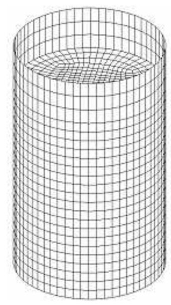

Fig.1 FEM model

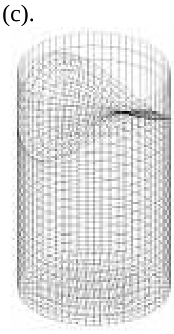

Fig.2 One of eigen modes

3.2 Seismic Response Analysis

Three direction seismic waves, whose acceleration response spectrums are shown in Fig.3, are input to the tank's bottom. The response is calculated for 20 seconds with a time step of 0.01 second. The CPU time of the method (b) is 2.3 times more than that of the method (a).

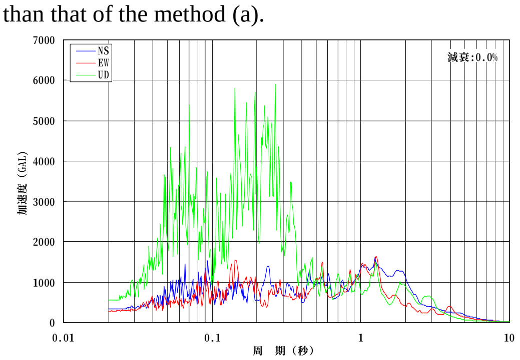

Fig.3 Acceleration response spectrums of seismic waves

4. CONCLUSIONS

The calculation cost of method (b) is higher than that of method (a) in eigen analysis and also higher than that of method (c) in dynamic response analysis. It is known that the reason is that sparsity is lost for the matrices M_f, $\mathbf{Q}_\eta$ and M_η. If the FEM model became larger than 10 thousand DOFs, the calculation time of the method (b) would be much longer than that of the method (a) and (c).

REFFERENCES

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