A practical 3D bounding surface plastic sand model for geotechnical earthquake engineering application

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Outline

- Brief review of 3 typical models:
  - **UBCSand, PM4Sand, DM04** (Dafalias-Manzari 2004 version)
  - All these three models are available at Itasca UDM website:

  https://www.itascacg.com/software/user-defined-constitutive-models-udm
- Formulation of the new model
- Performance of the new model

Evaluations of selected models

UBCSand
(Version 904aR)

PM4Sand
(Version 3)

DM04
(Version 2004)

1. Practice-friendly?
2. CRR-$(N_1)_{60}$ curve
3. CSR-N curves
4. Damping at large strain
5. Overlapped loop problem
6. Lode angle effect
7. Static or Dynamic?
8. Various densities need different parameters?
9. Kσ effect
10. Kα effect
11. Formula
12. General 3D model?

Red: Not Good; Green: Good; Black: Medium; Dash: Unknown.

Practice-friendly?

UBCSand | PM4Sand | DM04

- **UBCSand** & **PM4Sand**
  - Input parameters are easily calibrated from in-situ measurements
- **DM04**
  - Requires high quality lab test data for calibration
  - Unfortunately, the parameters calibrated from the lab-based sands usually cannot be directly used in practice for in-situ sands

CRR-$(N_1)_{60}$ curve

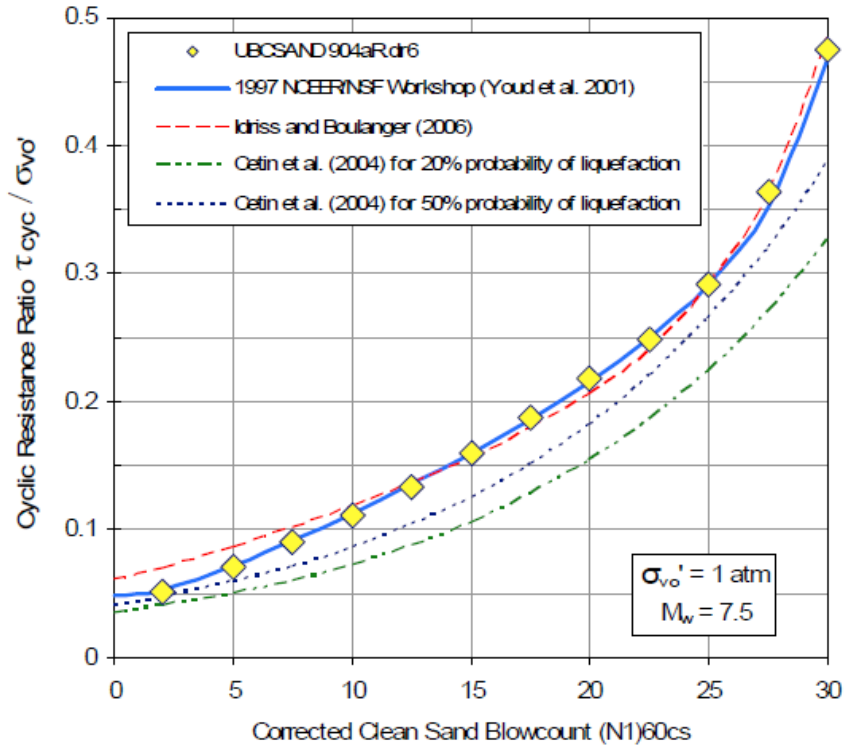
UBCSand | PM4Sand | DM04

- **UBCSand** is the first model matching it to the empirical curve.
- **PM4Sand** follows.
- **DM04** not matching.

CRRs indicated by the NCEER/NSF (Youd et al 2001) curve for a given corrected SPT blow-count, or $(N_1)_{60}$, should just induce liquefaction in an element if it is applied in 15 uniform cycles for an initial effective overburden stress of 1 atm during a DSS simulation.

⇕

CRRs at a M-7.5 earthquake.



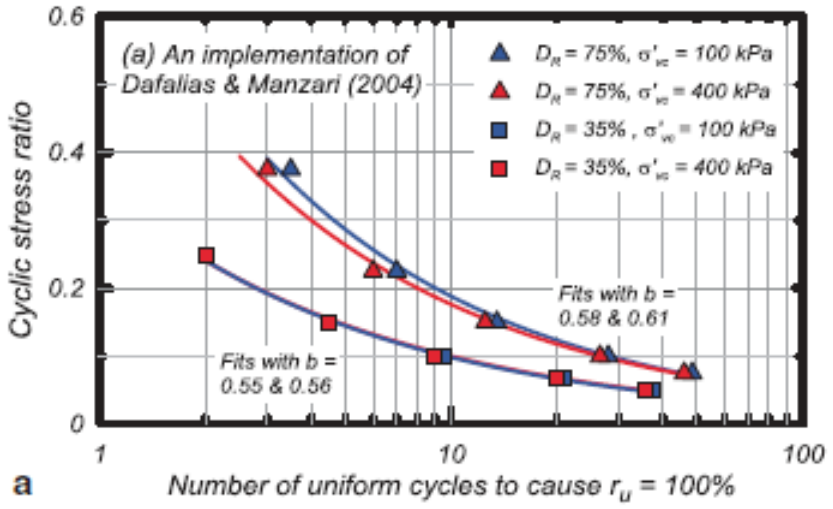
(After Beaty & Byrne, 2011)

CSR-N curves

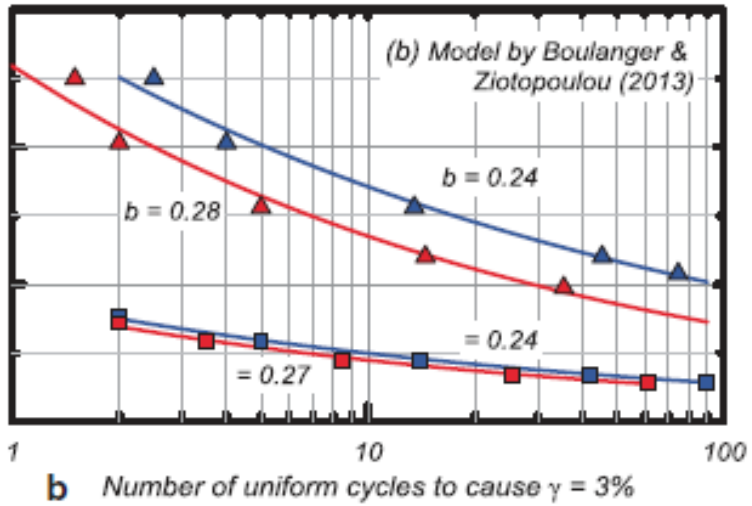
UBCSand | PM4Sand | DM04

- **DM04** may be too steep (b = 0.55 to 0.61).
- **PM4Sand** seems in the gentle side (b = 0.24 to 0.28) – heavily rely on lab-data.
- **UBCSand** seems in the most possible range for in-situ sands (b = 0.34 to 0.41).

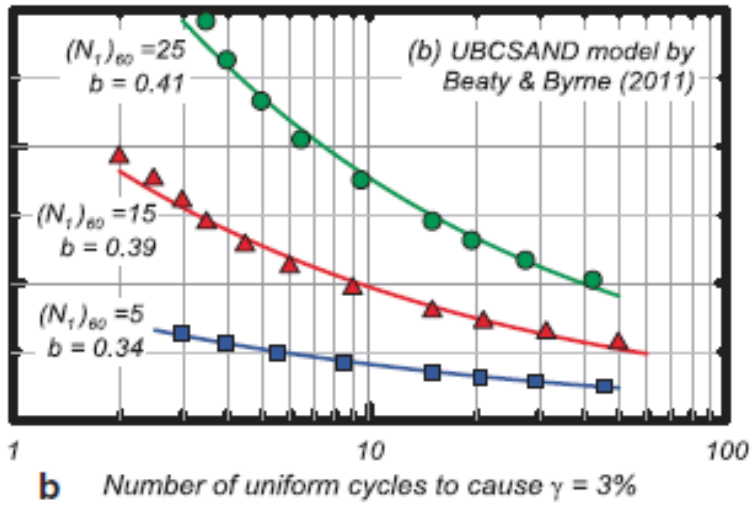
UBCSand



PM4Sand



SANISand



(After Boulanger, Montgomery, Ziotopoulou, 2017)

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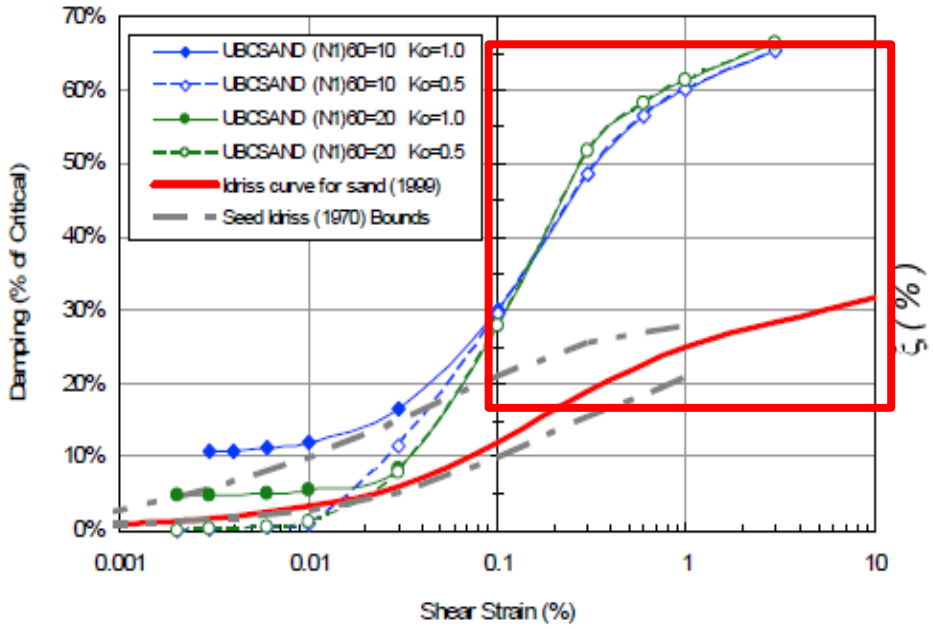
Damping at large strain

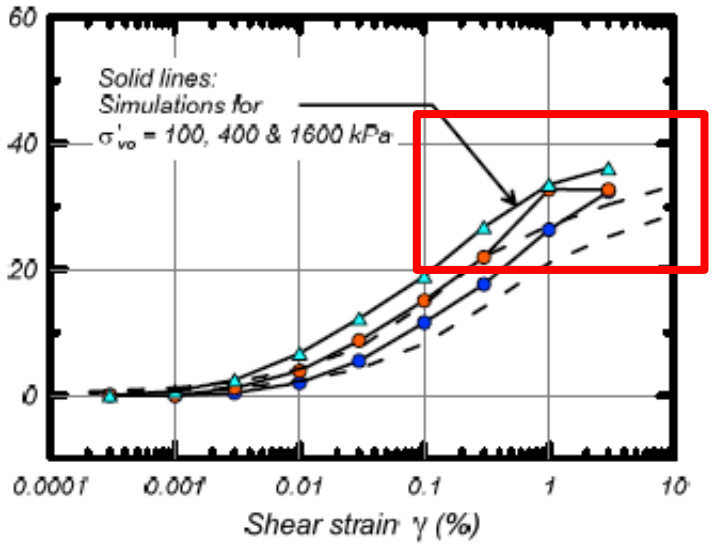
UBCSand | PM4Sand | DM04

UBCSand



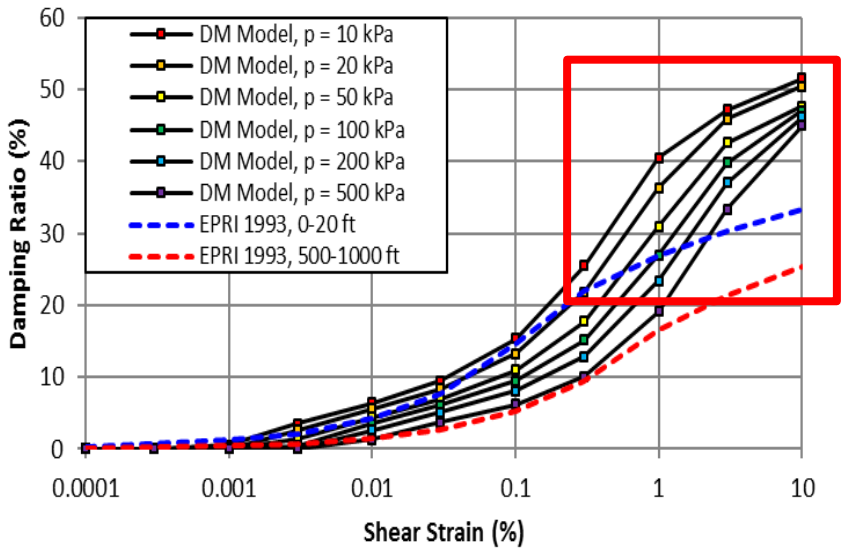
(After Beaty & Byrne, 2011)

PM4Sand



(After Boulanger & Ziotopoulou, 2015)

DM04



(After Cheng, Dafalias & Manzari, 2013)

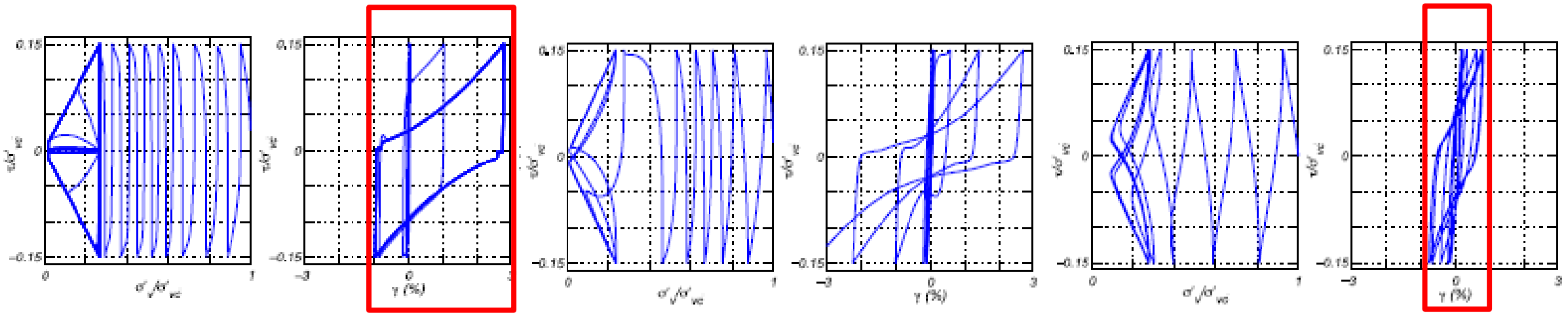
ITASCA™

Overlapped loop problem

UBCSand | PM4Sand | DM04

- **DM04** & **UBCSand** have overlapped loops and no more accumulated shear strains thereafter.

UBCSand **PM4Sand** **DM04**



-1% ~ 3%

-1% ~ 1%

(After Carey & Kutter, 2017)

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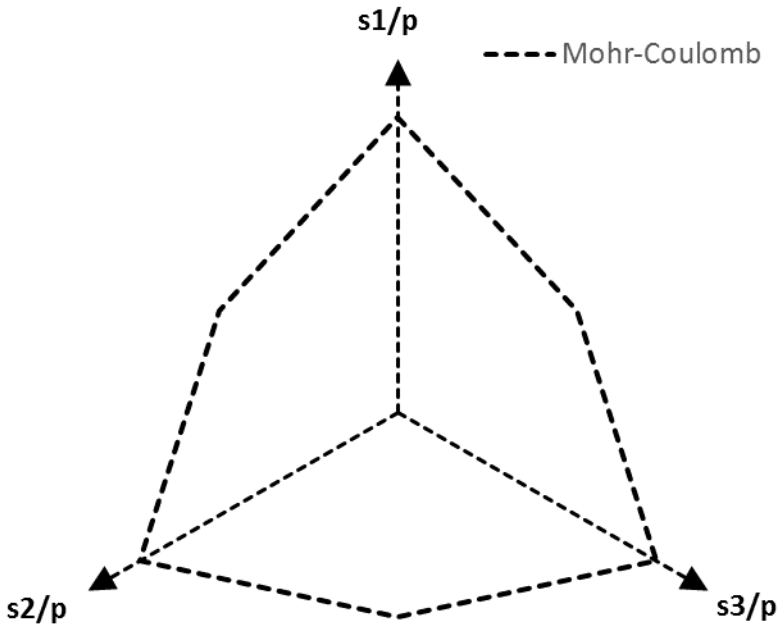
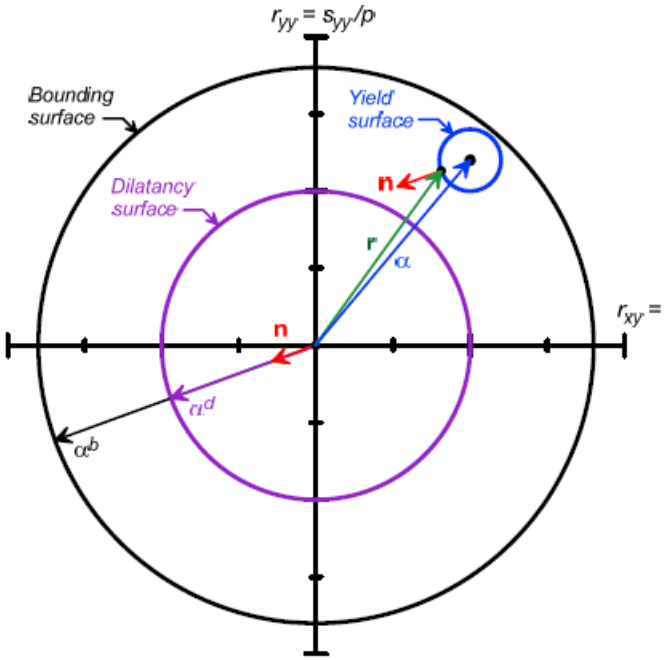
Lode angle effect

UBCSand | PM4Sand | SANISand

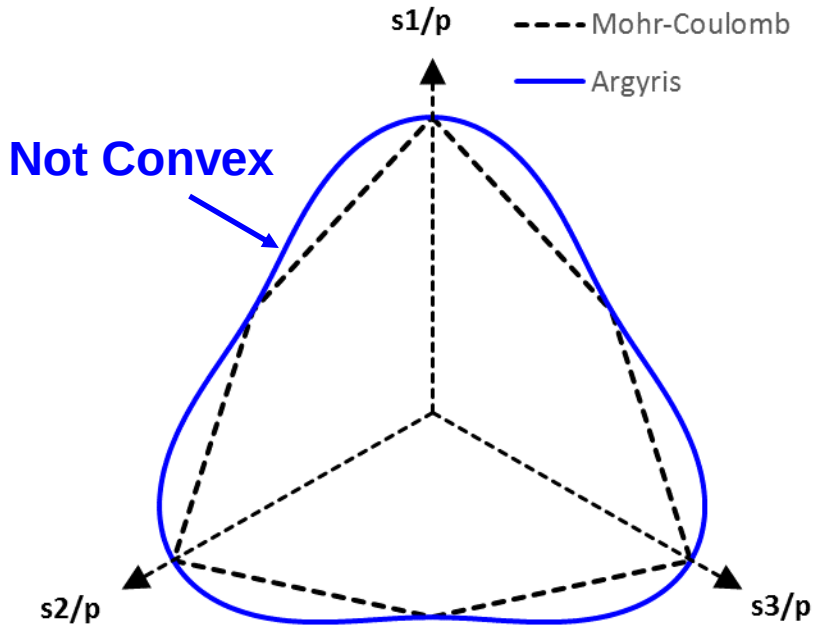
UBCSand



PM4Sand



DM04



(After Boulanger & Ziotopoulou, 2015)

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Static?
Monotonic?

UBCSand | PM4Sand | DM04

UBCSand:

- “A new parameter m_static was added to permit the model to function in a simpler manner when used during pre-earthquake static analyses.” (Beaty & Byrne, 2015).

PM4Sand:

- “PM4Sand has been validated for use with the dynamic procedure only.”
- “If the monotonic behavior was more important than the CRR values, then a different calibration would be required.” (Boulanger & Ziotopoulou, 2015).

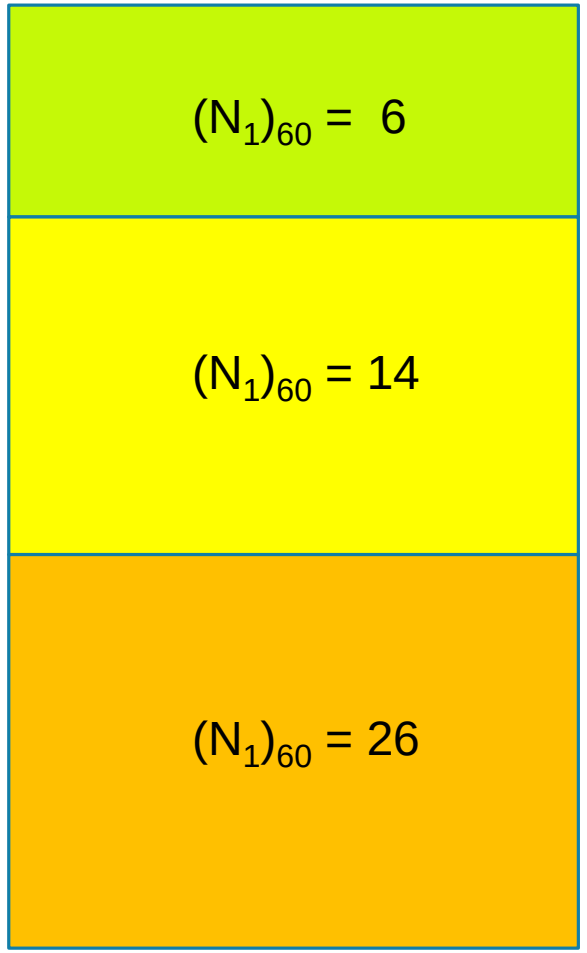
DM04:

- Excellent performance on monotonic loading/unloading. (Dafalias & Manzari, 2004)

Various densities need different parameters?

UBCSand | PM4Sand | DM04

Material #	UBCSand / PM4Sand	DM04
Sand #1	Parameter set #1	One set for all
Sand #2	Parameter set #2	
Sand #3	Parameter set #3	

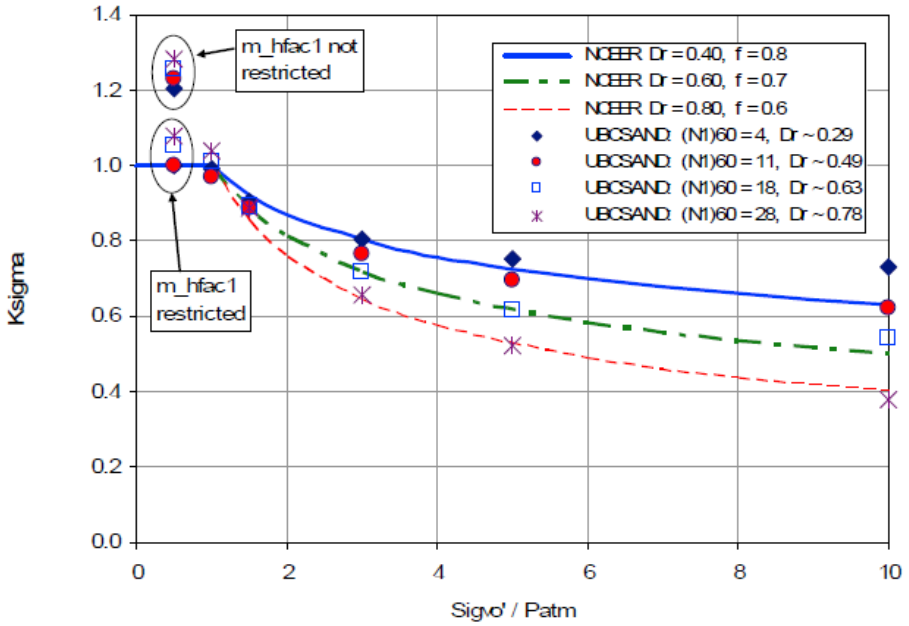


Kσ effect

UBCSand | PM4Sand | DM04

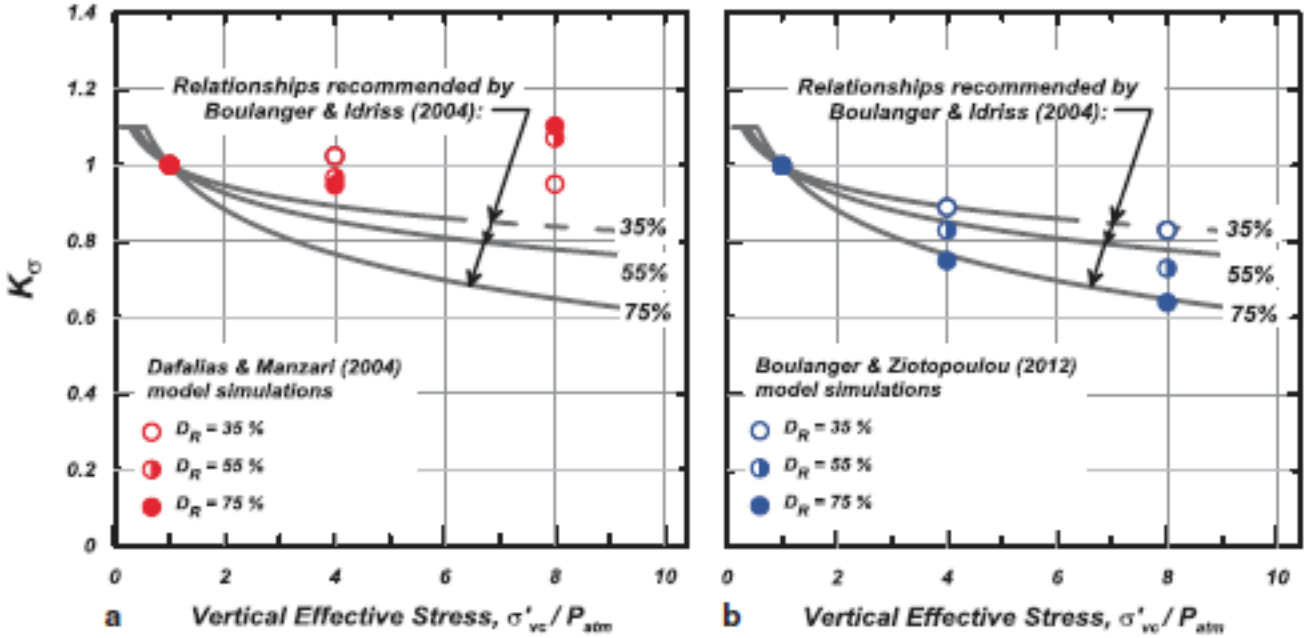
$$K_\sigma = \frac{CRR_{\sigma'_{v0}}}{CRR_{\sigma'_{v0}=1\ atm}}$$

UBCSand



(After Beaty & Byrne, 2011)

DM04



(After Boulanger, Montgomery, Ziotopoulou, 2017)

Kα effect

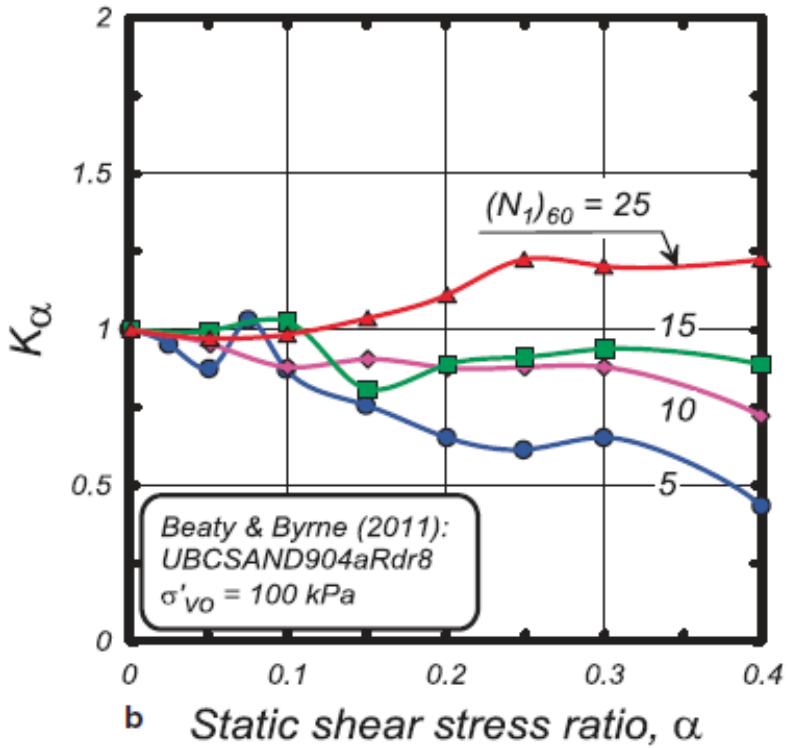
UBCSand | PM4Sand | DM04

$$K_\alpha = \frac{CRR_\alpha}{CRR_{\alpha=0}}$$

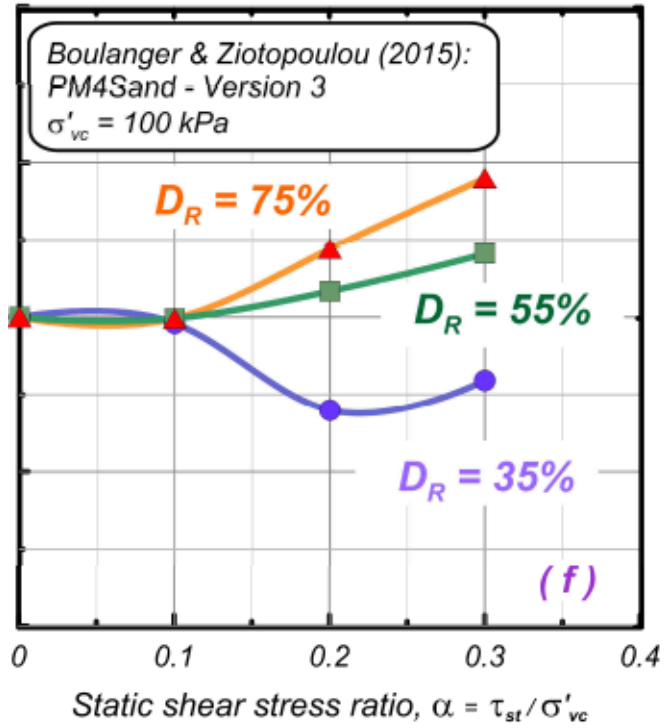
$$\alpha = \frac{\tau_{static}}{\sigma'_{v0}}$$

Not recommended for routine practice. (NCEER/NSF Workshop, Youd et al 2001 & NASEM, 2016)

UBCSand



PM4Sand



(After Ziotopoulou & Boulanger, 2015)

Formula

UBCSand PM4Sand DM04

	UBCSand	PM4Sand	DM04
Difficulty	Relatively simple	Overly complex and lengthy	Relatively simple
Documentation	More details required	Well documented	Well documented

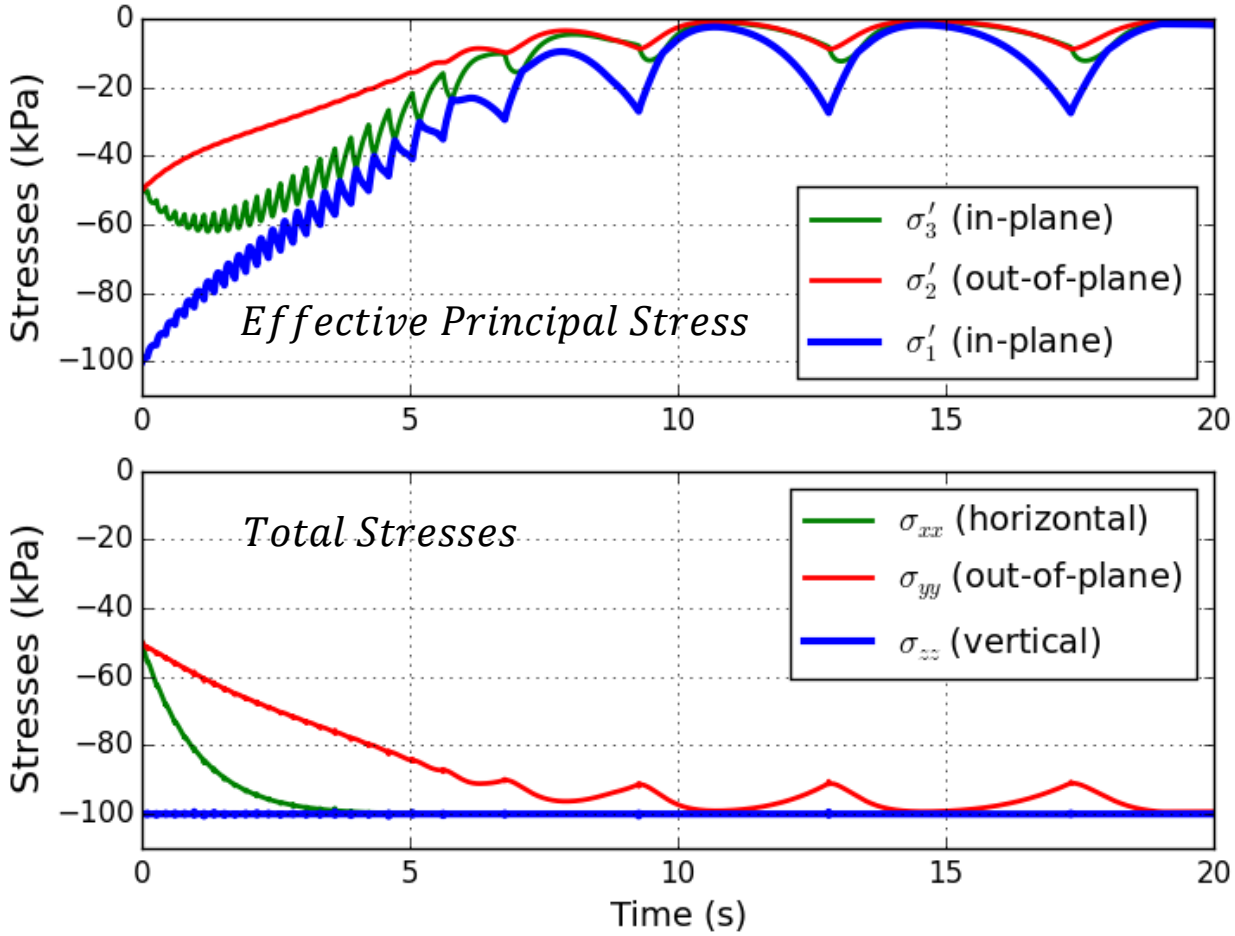
General 3D ?

UBCSand | PM4Sand | DM04

UBCSand	2D Plane-Strain
PM4Sand	2D Plane-Strain
DM04	General 3D

Even for a 2D plane-strain model, the out-of-plane stress should be in formulated and documented!

Cyclic DSS using 3D model:



Evaluation Summary

Note: Red: Not Good; Green: Good; Black: Medium

	UBCSAND (Version 904aR)	PM4SAND (Version 3)	DM04 (Version 2004)
Practice-friendly ?	**Yes**	**Yes**	**No**
CRR-$(N_1)_{60}$ curve	**Match semi-empirical**	**Match semi-empirical**	**Not matching**
CSR-N curves	**Ok**	**OK, maybe in the gentle side**	**Overly steep**
Damping at large strain	**Overly large damping**	**Ok**	**Overly large damping**
Overlapped loop problem	**Yes**	**No**	**Yes**
Lode angle effect	**Same as MC model**	**No**	**Yes, but not convex**
Static ? **Monotonic ?**	**Depends**	**Not for static;** **Need different calibration**	**OK**
Various densities	**Need different calibration**	**Need different calibration**	**One set of parameters**
Kσ effect	**Empirical match**	**Empirical match**	**Not matching**
Kα effect **(*not suggested for design*)**	**Trend OK**	**Trend OK**	**Not matching**
Formula difficulty **Formula documentation**	**Relatively simple** **Not well documented**	**Overly complex & lengthy** **Well documented**	**Relatively simple** **Well documented**
General 3D model?	**No, only for plain-strain**	**No, only for plain-strain**	**Yes**

The New model

- Based on **DM04** formula
- Targets are to overcome the aforementioned shortcomings (red → green)
  - ❖ General 3D
  - ❖ As simple as possible
  - ❖ Practice-friendly
  - ❖ Matches to the empirical relations

DM04 (Version 2004)	New Model
No	Green
Not matching	
Overly steep	
Overly large damping	
Yes	
Yes, but not convex	
OK	
One set of parameters	
Not matching	
Not matching	
Simple Well documented	
Yes	

Key modifications

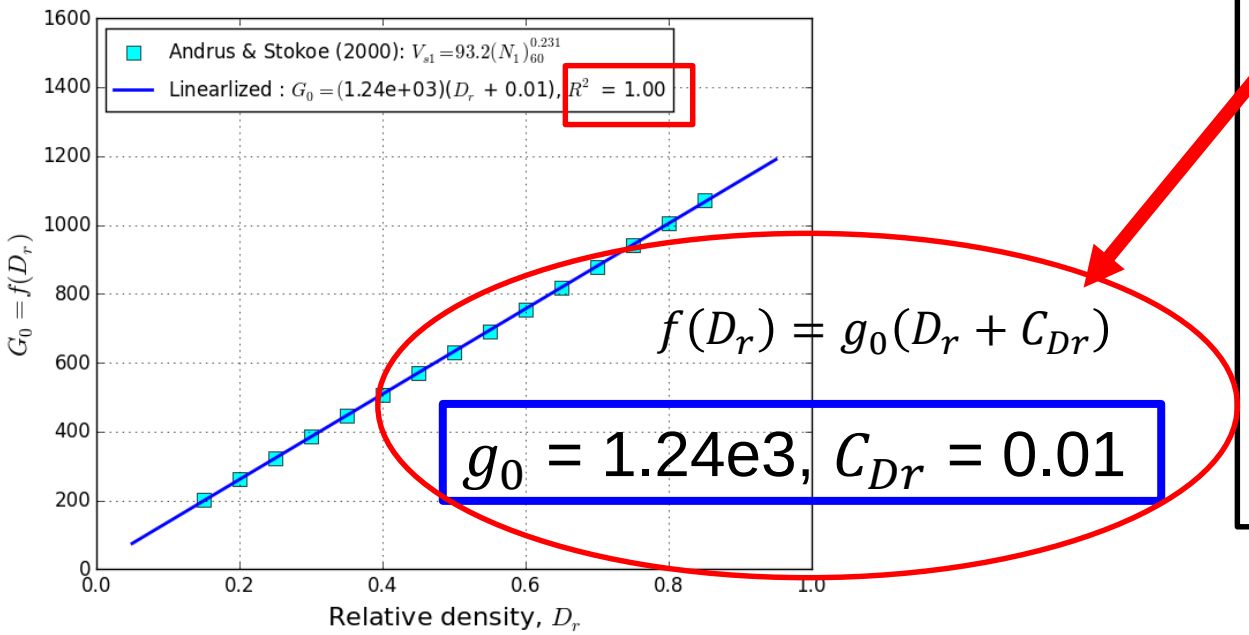
- **DM04** is a **Void-Ratio (e)** based model.
- **New model** is a **Relative-Density (D_r)** based model, because D_r can be easily and reliably calibrated from $(N_1)_{60}$ or (q_{c1N}), e.g.,

$$D_r = \sqrt{\frac{(N_1)_{60}}{46}}$$

$$D_r = 0.268 \ln q_{c1N} - b_x$$

Elastic moduli

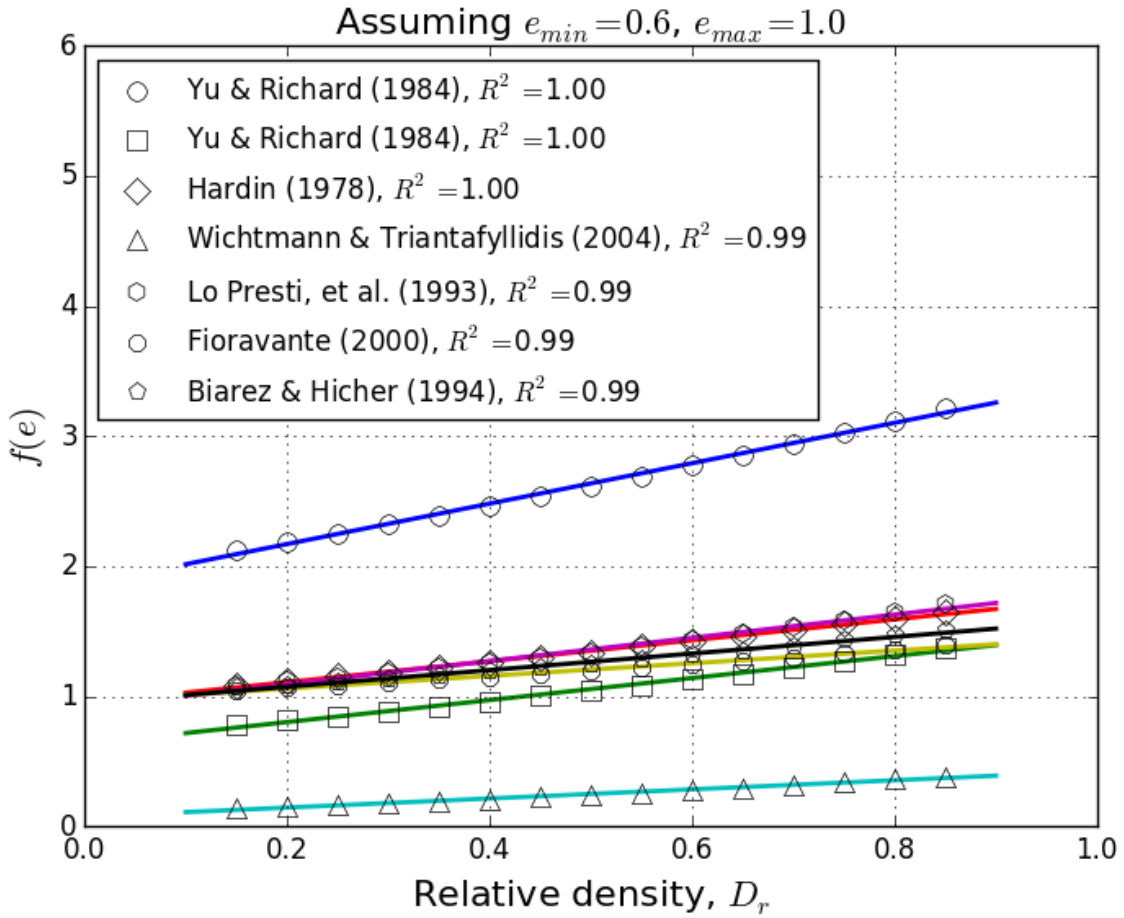
- **DM04**: $G = f(e)P_{atm}(\frac{p}{P_{atm}})^{0.5}$, $f(e) = 250\frac{(2.97-e)^2}{1+e}$
- **This model**: $G = f(D_r)P_{atm}\left(\frac{p}{P_{atm}}\right)^{0.5}$



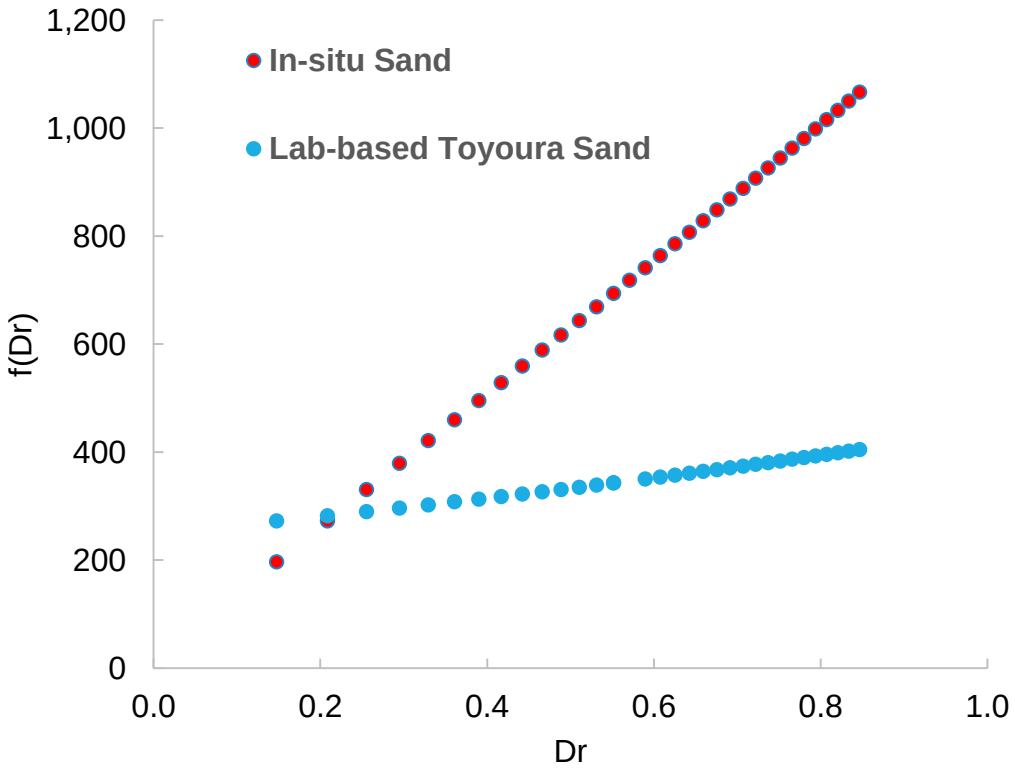
1. Andrus & Stokoe (2000): $V_{s1} = 93.2(N_1)_{60}^{0.231}$
2. $V_s = V_{s1}\left(\frac{P_{atm}}{\sigma'_{vo}}\right)^{0.25}$
3. $G = \rho_{sat}V_s^2$
4. $\rho_{sat} = \frac{(G_s+e)\rho_w}{1+e}$, , typical $G_s = 2.65$
5. $p = \frac{(1+2K_0)}{3}\sigma'_{vo}$, typical $K_0 = 0.5$

$$f(D_r) = \frac{\rho_{sat}}{P_{atm}}\sqrt{\frac{3}{(1+2K_0)}V_{s1}^2}$$

Elastic moduli



$$G = f(D_r)P_{atm}\left(\frac{p}{P_{atm}}\right)^{0.5}$$



Lab-based sand data support the linear relation too!

Modified/New Formula

- **Elasticity** $[G = g_0(D_r + C_{Dr})P_{atm}\left(\frac{p}{P_{atm}}\right)^{0.5}]$
- Critical State [2 or 3-parameter in terms of Dr]
- Lode's angle dependence [Eekelen locus, always convex]
- State Parameter $[I_r = D_r \ln\left(\frac{p}{p_c}\right)]$
- Bounding/Dilatancy Surfaces […]
- Plastic Modulus […]
- Plastic Volumetric Strain […]
- Fabric-Dilatancy Tensor […]
- Stiffness Damage [new]

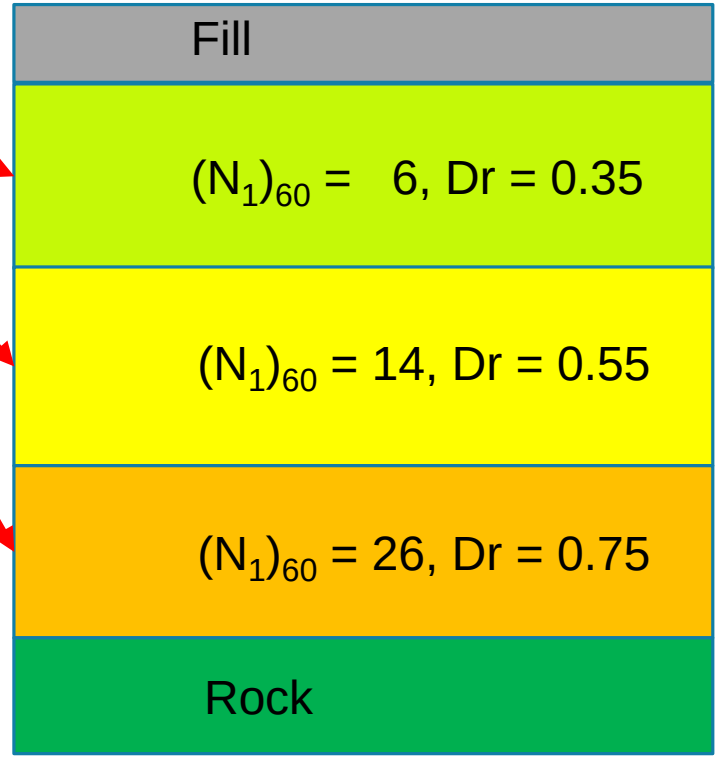
- Not just $D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$
- Formula relatively simple
- Every modification has its reason

Practice-friendly

As simple as:

```
zone property relative-density-initial 0.35 range group 'LooseSand'
zone property relative-density-initial 0.55 range group 'MedianSand'
zone property relative-density-initial 0.75 range group 'DenseSand'

# input initial stress components as initial-condition parameters
…
```



All other parameters are defaults or internally-calibrated.

This model has been internally calibrated to match NCEER/NSF (Youd et al, 2001) CRR curve for clean sands.

Practice-friendly, to refine?

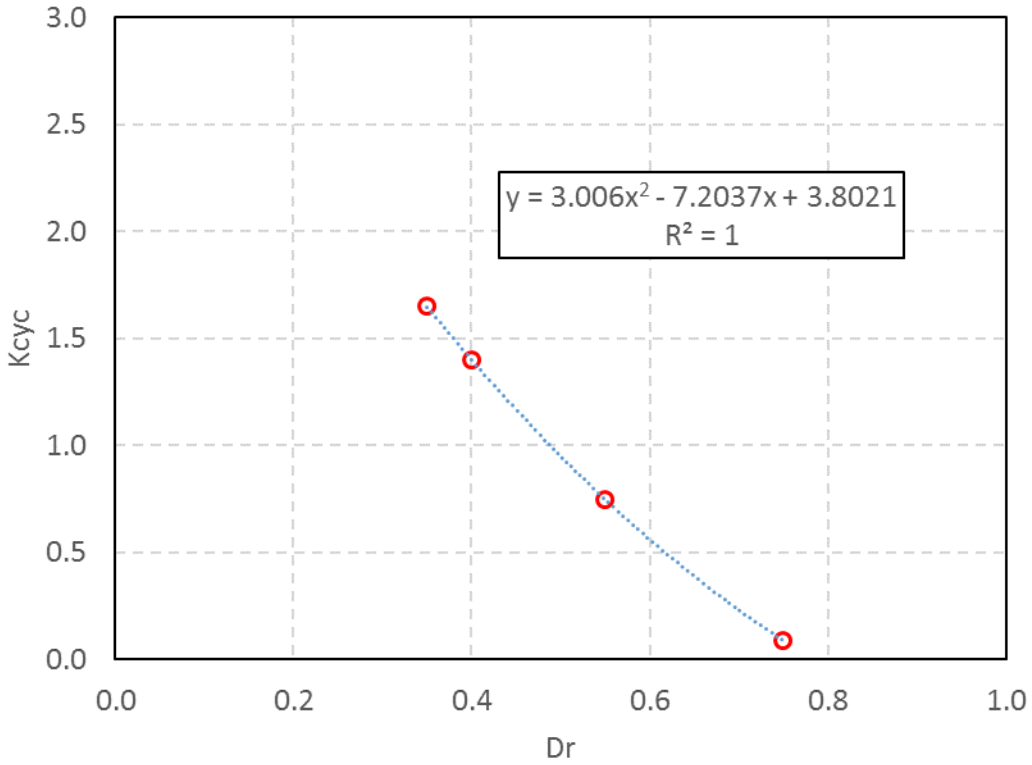
If you wish to refine, e.g.,

$\phi_{cv} = 36$ (degrees)

$e_{max} = 0.8$, $e_{min} = 0.5$

$Q = 9.5$, $e_{max} = 0.9$

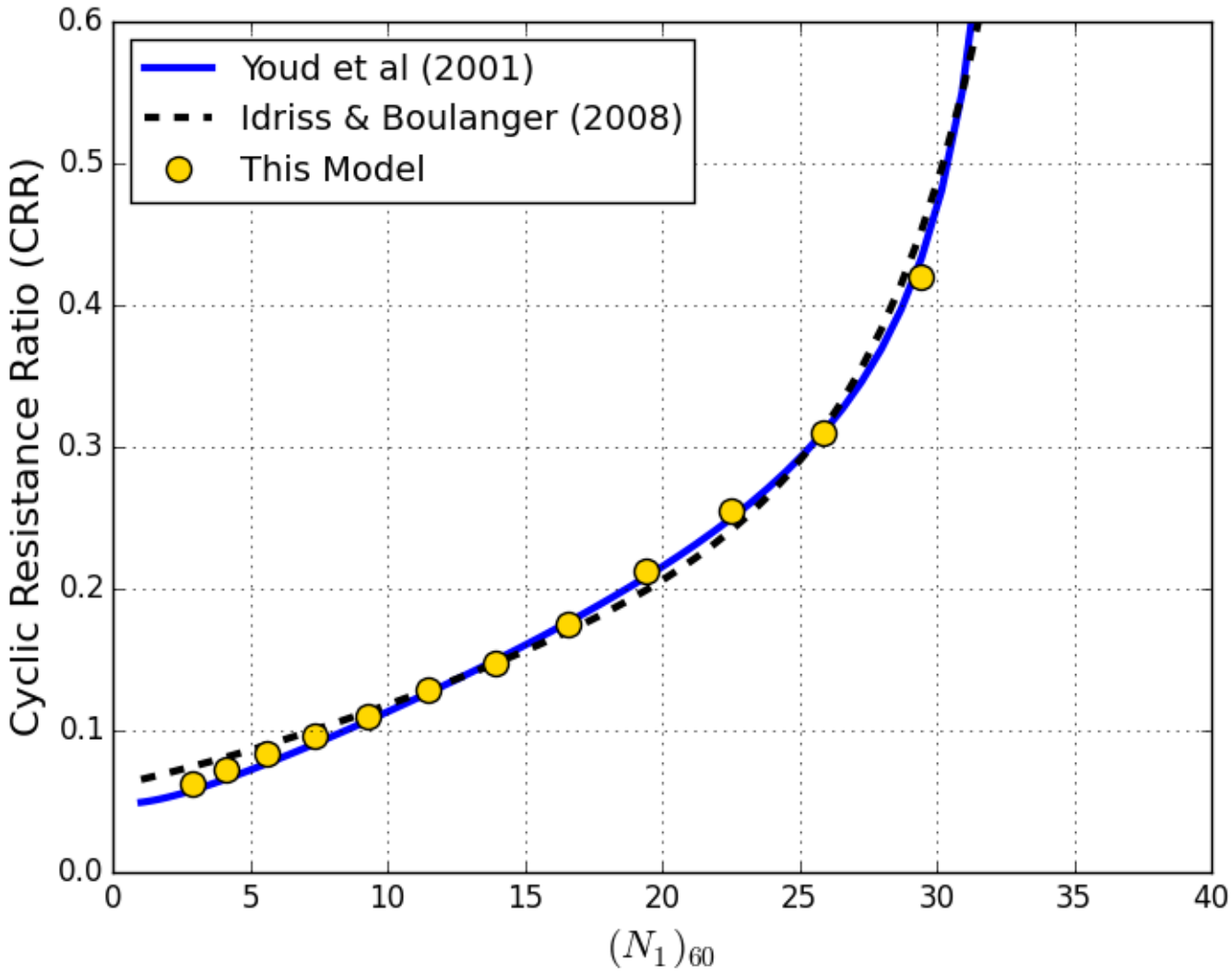
- (Optional) Use an spread sheet to estimate g_0 and C_{Dr};
- (Usually not needed) Other refinements if you wish;
- K_{cyc} should be calibrated lastly:
  - Try 3 or more single-zone DSS simulations with various relative densities to obtain 3 or more pairs (D_r, K_{cyc}) to match the target CRRs.
  - Plot the pairs in an Excel sheet to fit with a **quadratic** curve, to see if with a satisfactory correction or not:
    - If Yes, input (A_0, A_1, A_2);
    - If No, input the pairs as a table-type parameter.



$$K_{cyc} = A_0 - A_1 D_r + A_2 D_r^2$$

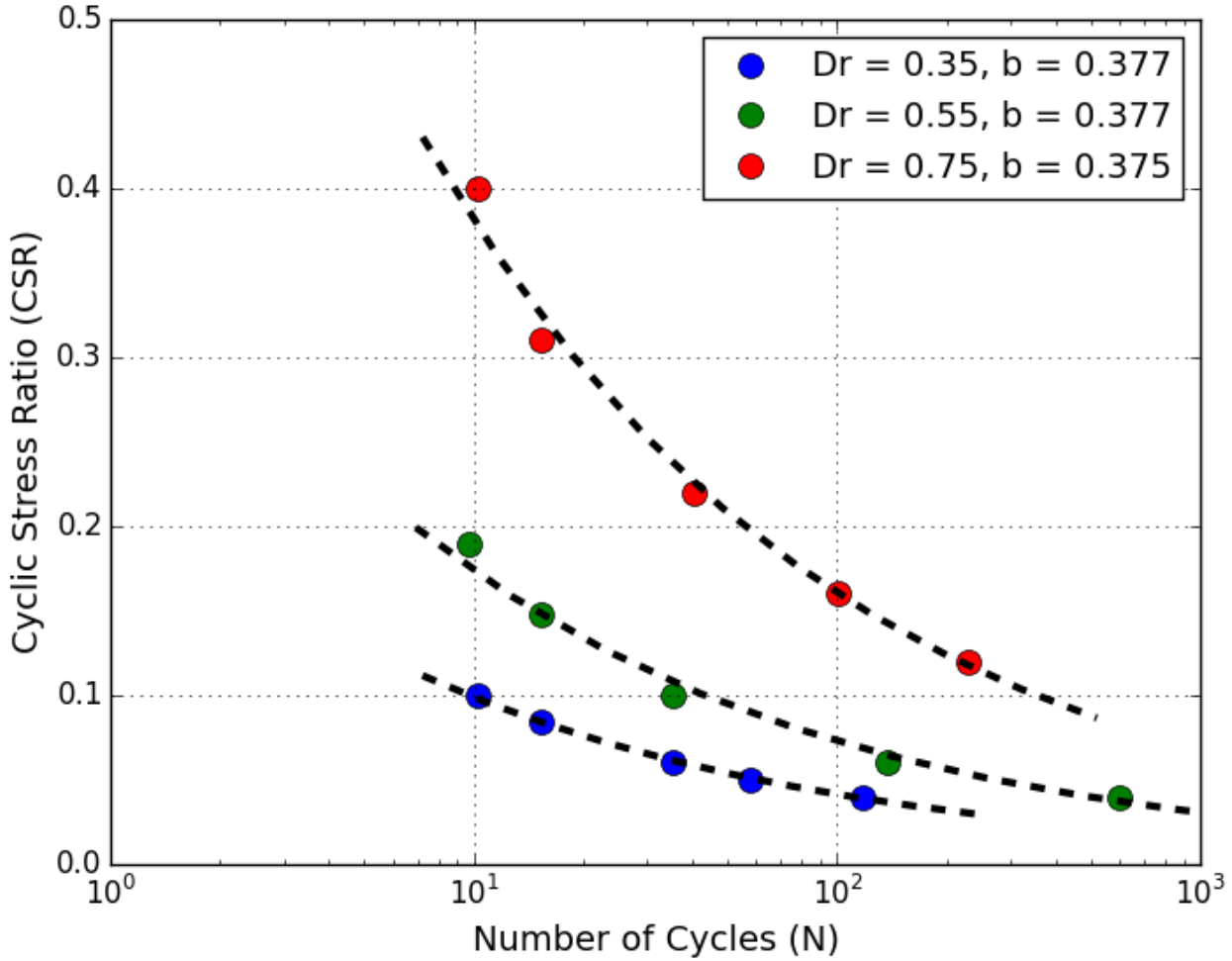
(A_0, A_1, A_2) = (3.8, -7.2, 3.0)

CRR-$(N_1)_{60}$

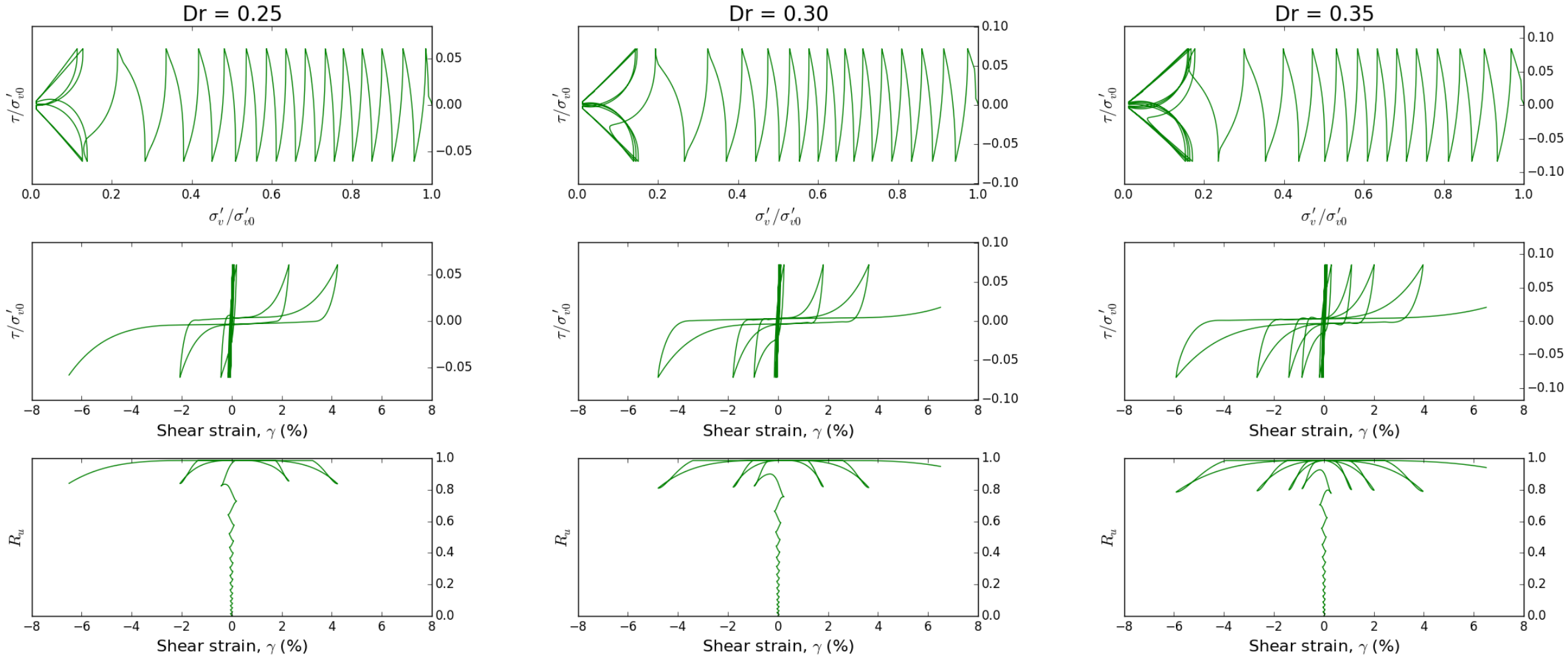


Liquefaction in 15±0.25 uniform cycles versus $(N_1)_{60}$, DSS numerical tests at initial vertical effective stress 1 atm & K_0 = 0.5.

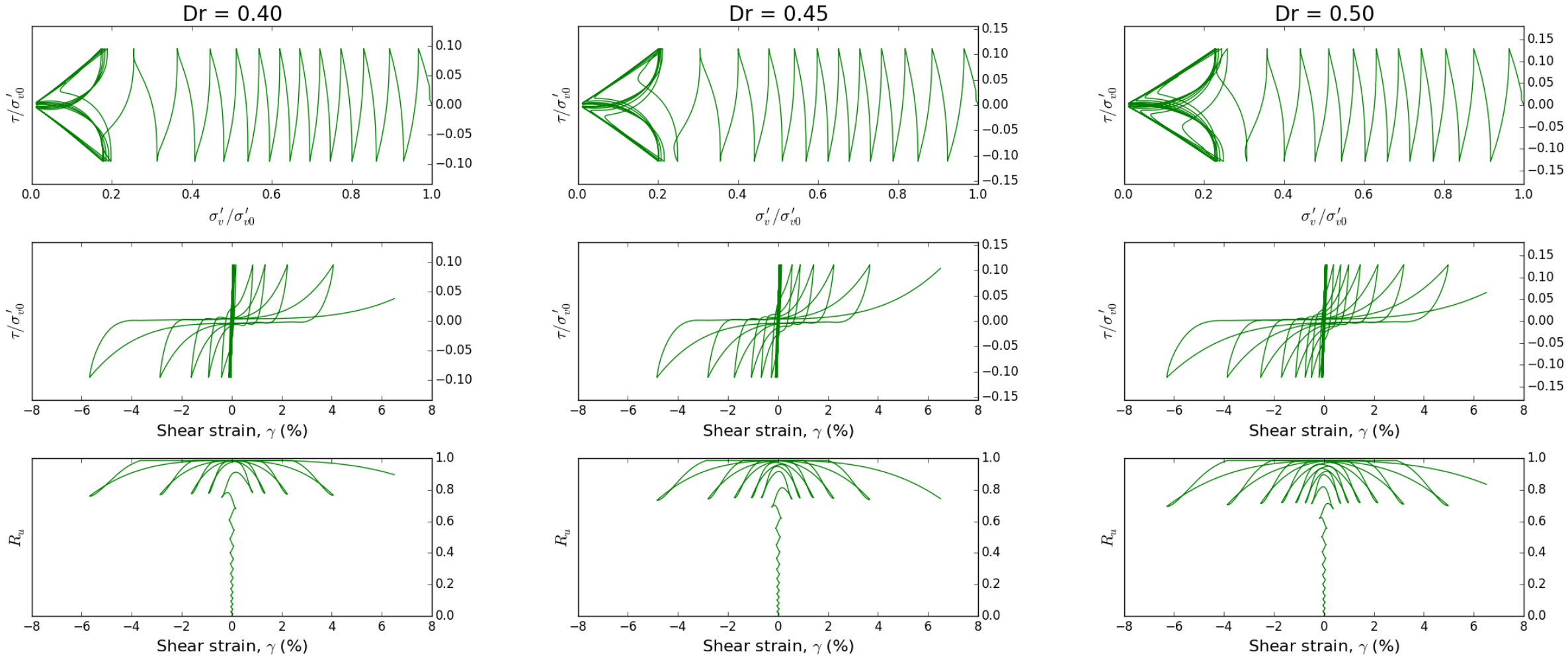
CSR-N


Dr = 0.35, b = 0.377
Dr = 0.55, b = 0.377
Dr = 0.75, b = 0.375
Cyclic Stress Ratio (CSR)
Number of Cycles (N)
0.0
0.1
0.2
0.3
0.4
0.5
10^0
10^1
10^2
10^3

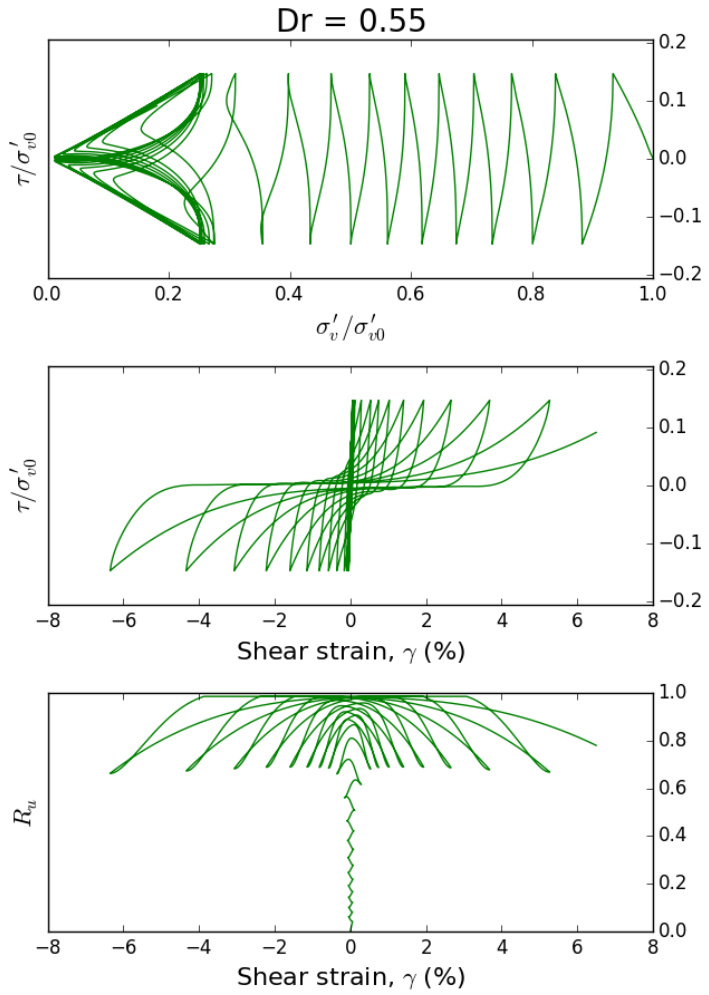
DSS cyclic simulation

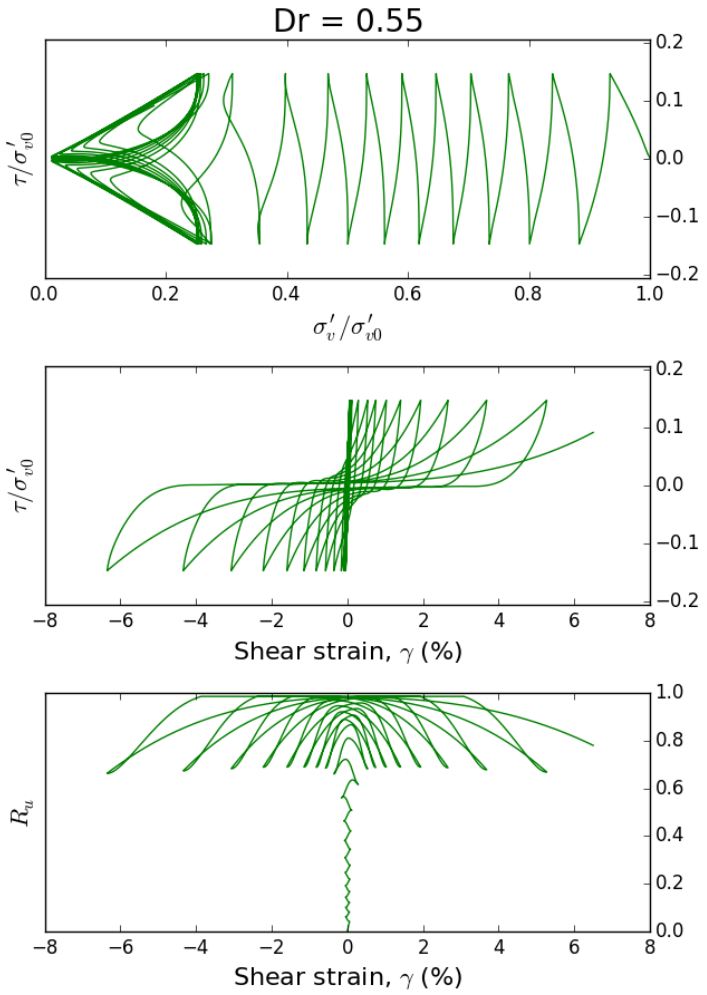


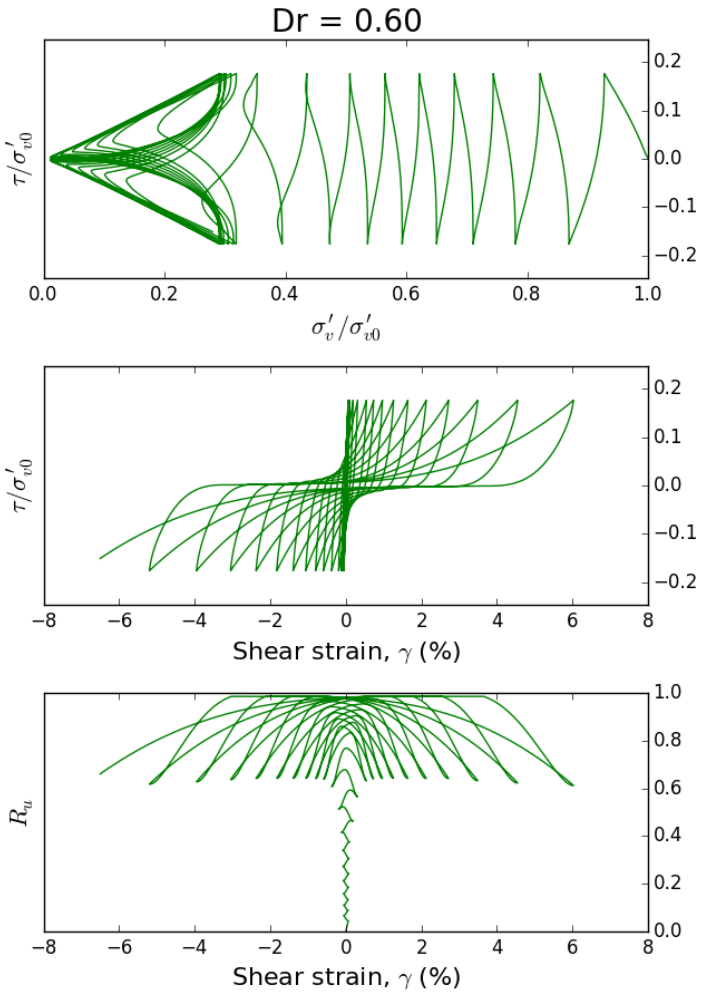
DSS cyclic simulation



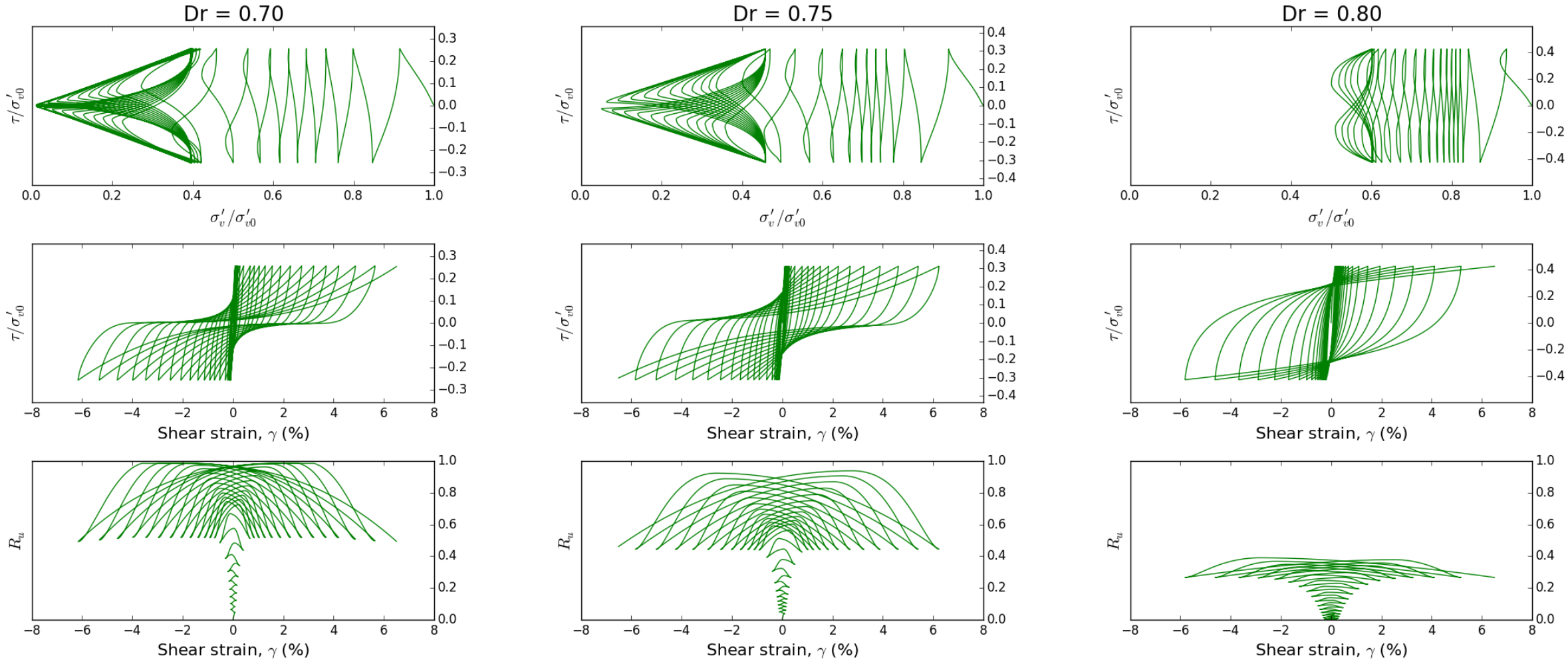
DSS cyclic simulation







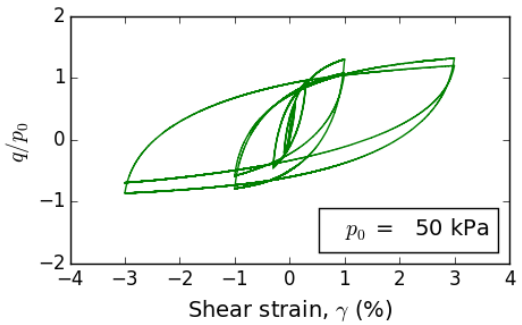
DSS cyclic simulation

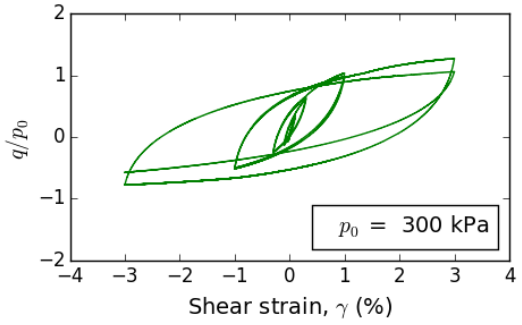


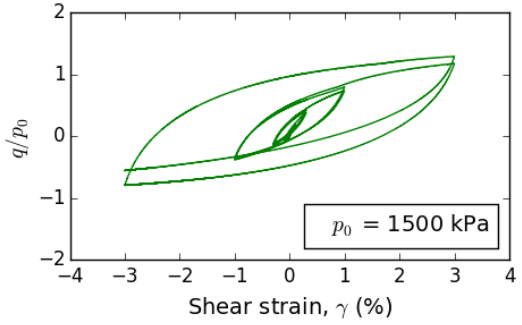
G/G_{max} & damping

constant-p triaxial test simulation

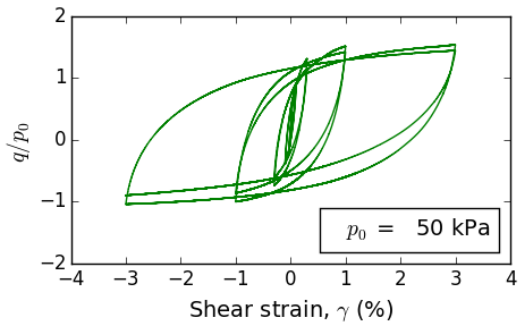
Dr = 0.35

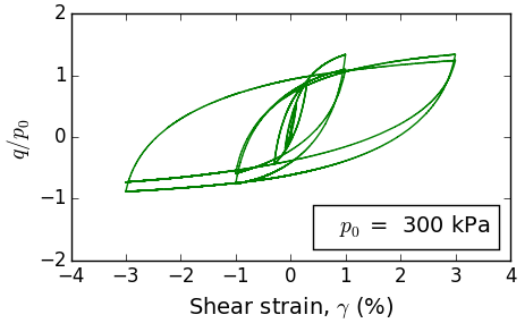


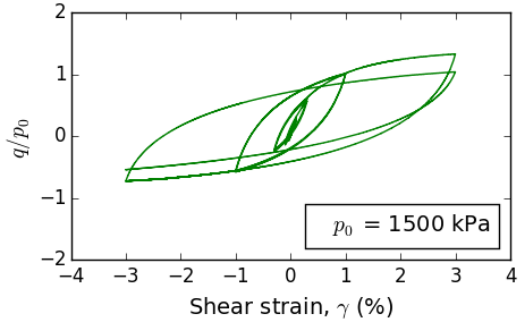




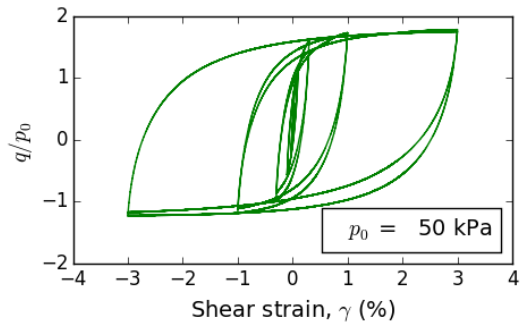
Dr = 0.55

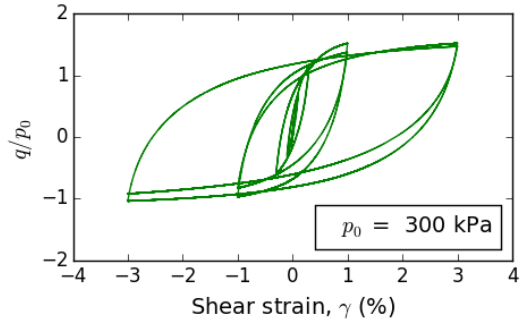


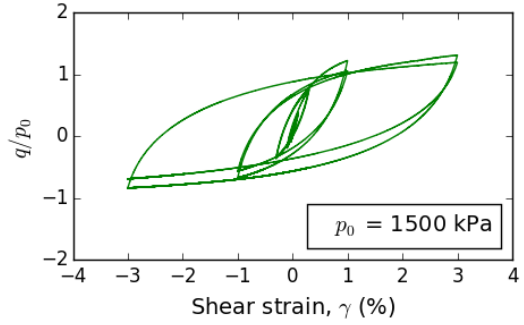




Dr = 0.75



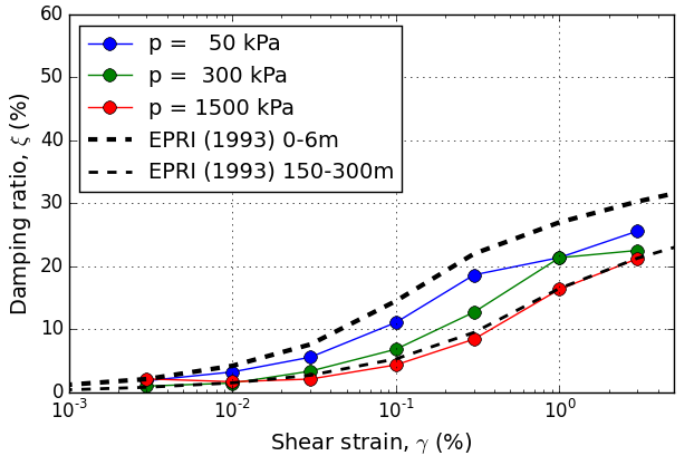


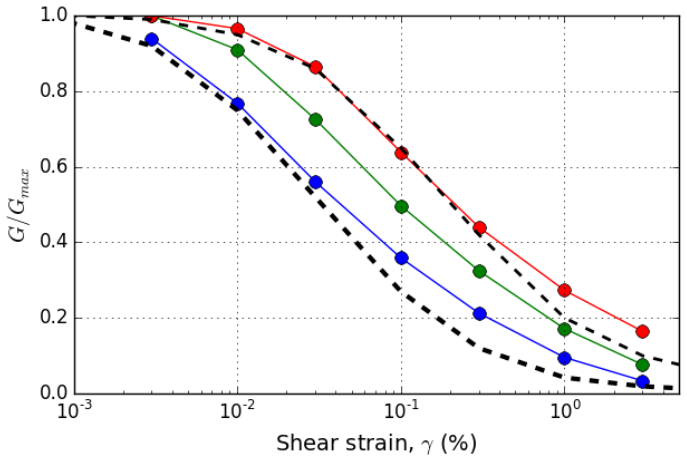


G/G_{max} & damping

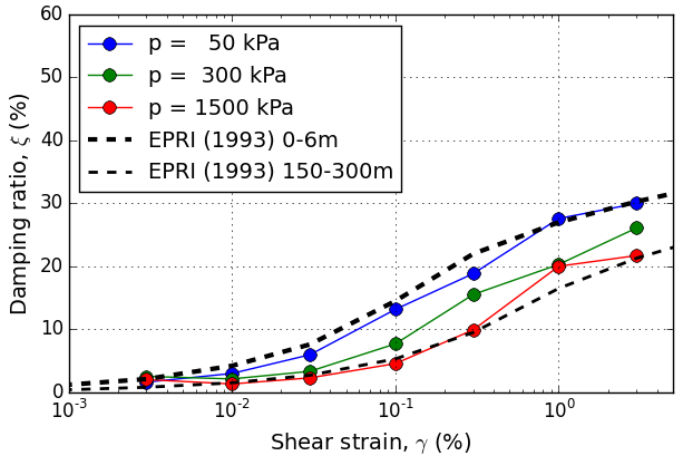
constant-p triaxial test simulation

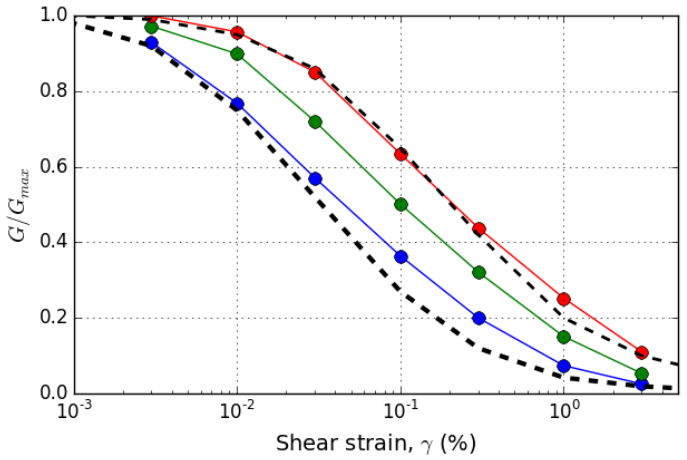
Dr = 0.35



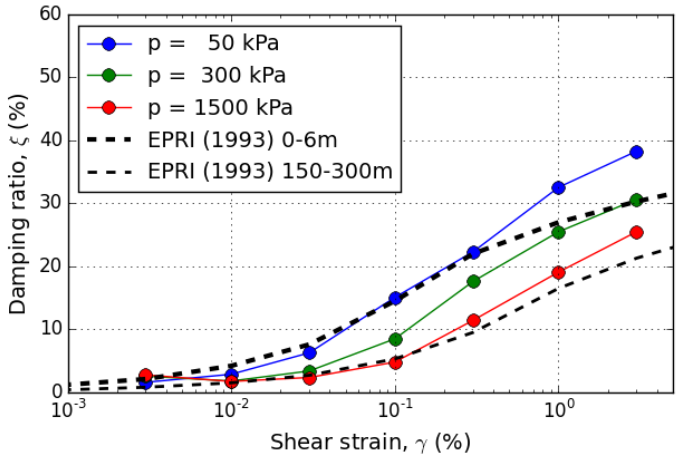


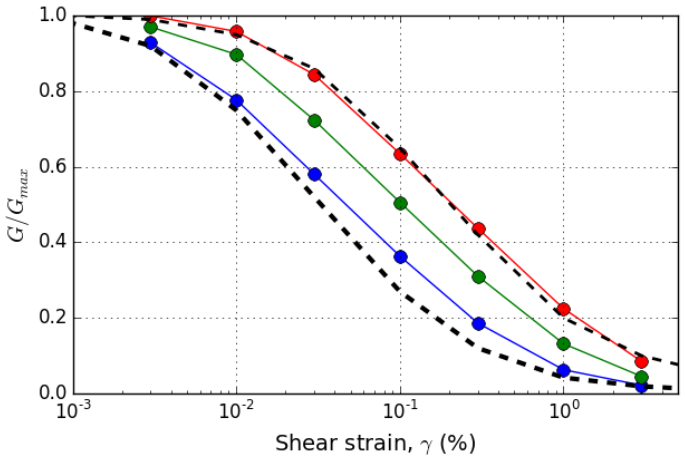
Dr = 0.55



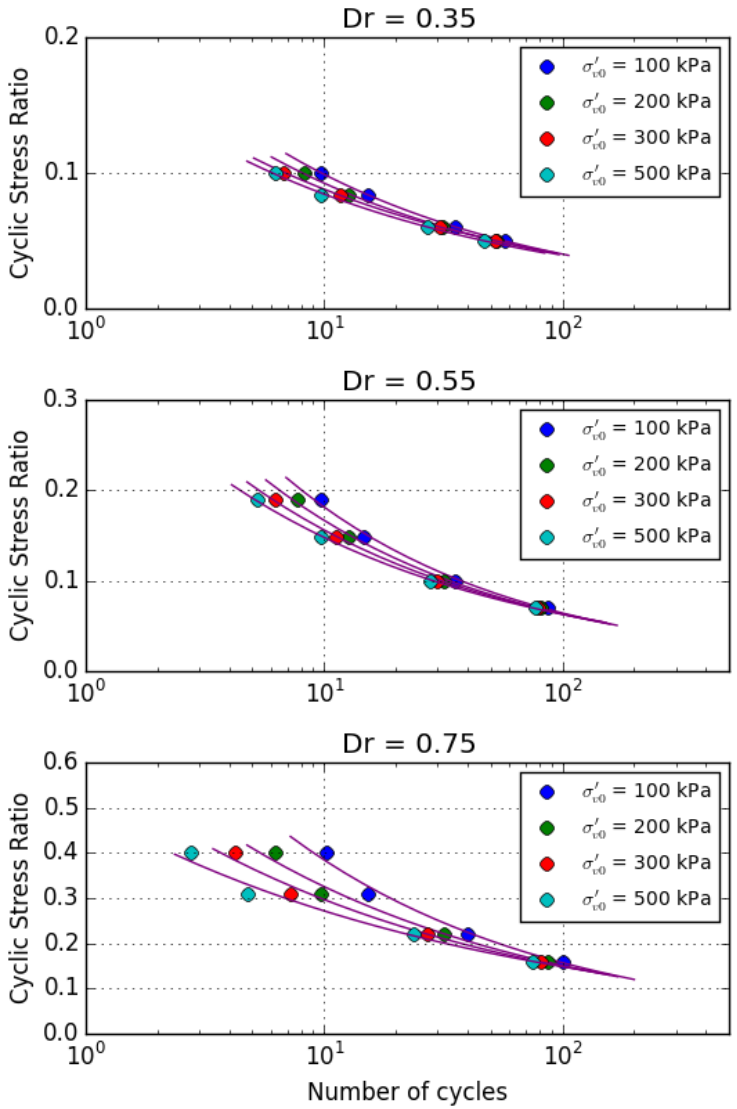


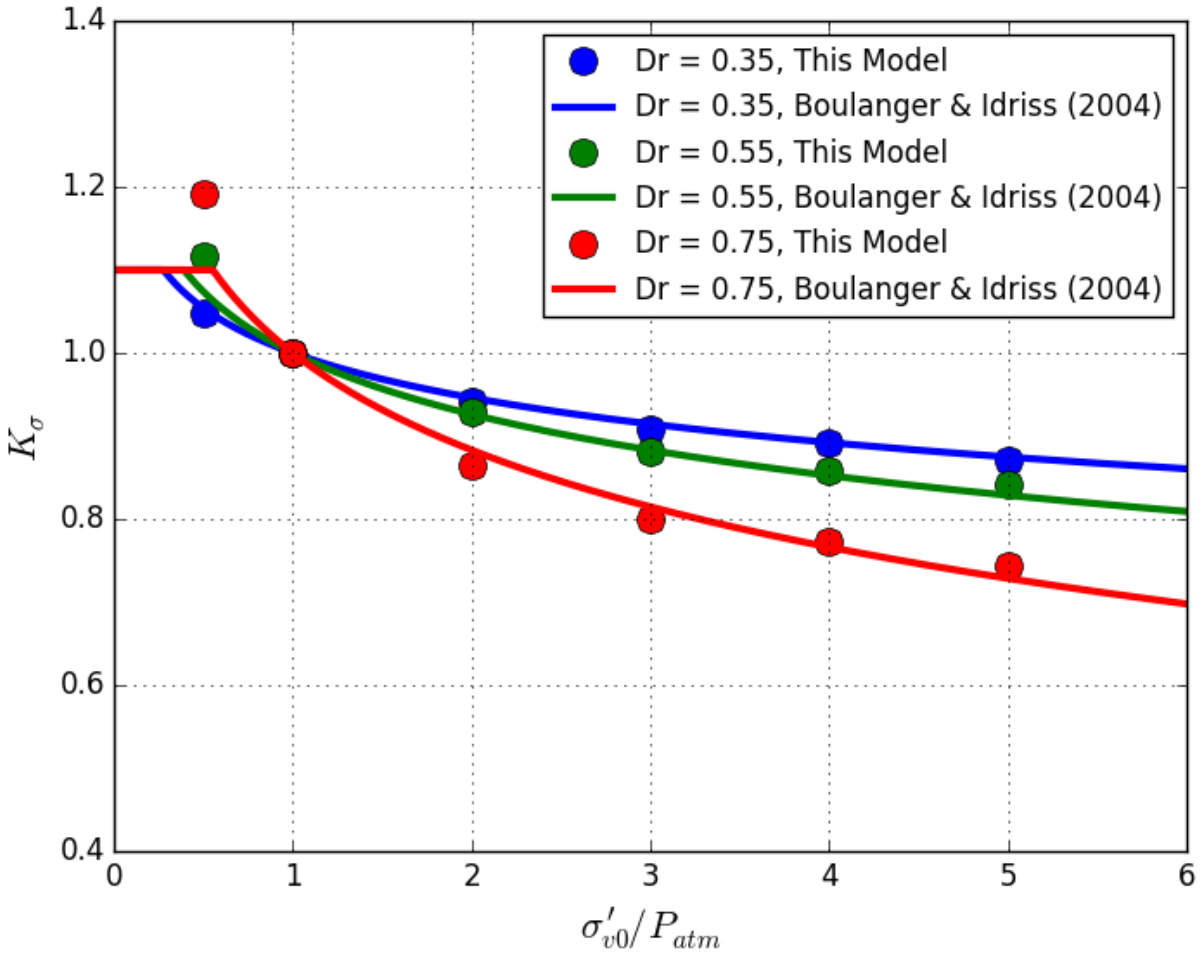
Dr = 0.75





Kσ effect





For lab-based sands?

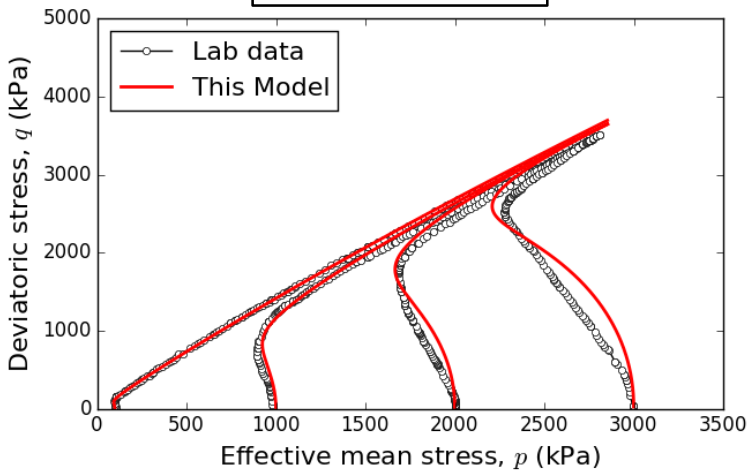
- All default and internally-calibrated parameters are for in-situ sands.
- For lab-based sands, more calibration efforts required.

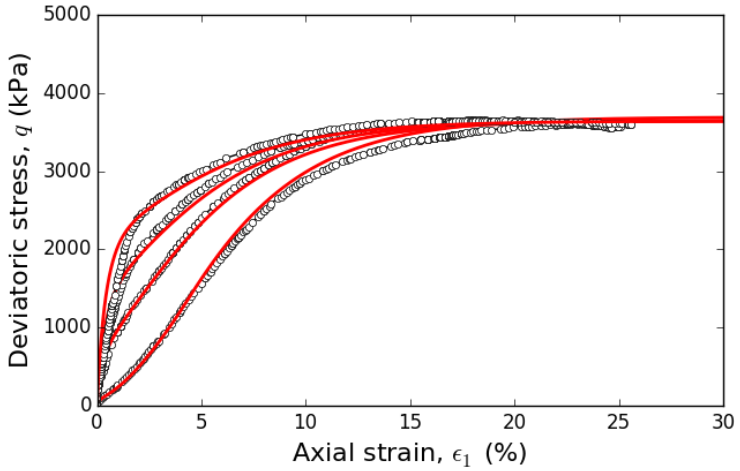
Lab-based Toyoura sands	Model defaults
$\phi_{cv} = 31.7$	$\phi_{cv} = 33$
$g_0 = 200$, $C_{Dr} = 1.12$	$g_0 = 1.24e3$, $C_{Dr} = 0.01$
$D_{rc0} = 0.115$, $\lambda_r = 0.05$, $\xi = 0.7$	$Q = 10$, $R = 1$
(3-parameter critical-state equation)	(2-parameter critical-state equation)

```
zone property friction-critical 31.7
zone property elasticity-1 200 elasticity-2 1.12
zone property critical-state-1 0.115 critical-state-2 0.05 critical-state-3 0.7
```

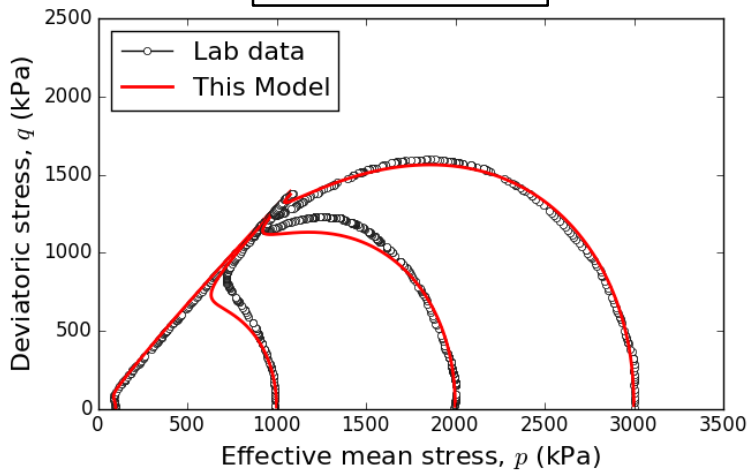
Toyoura sand
Monotonic undrained triaxial simulation

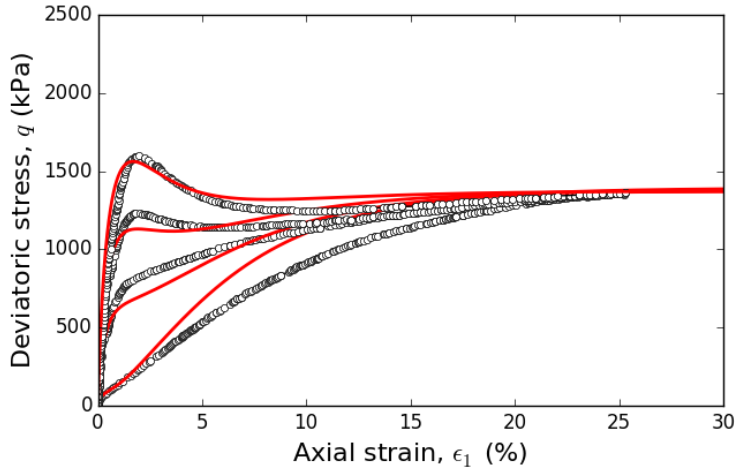
Dr = 0.637



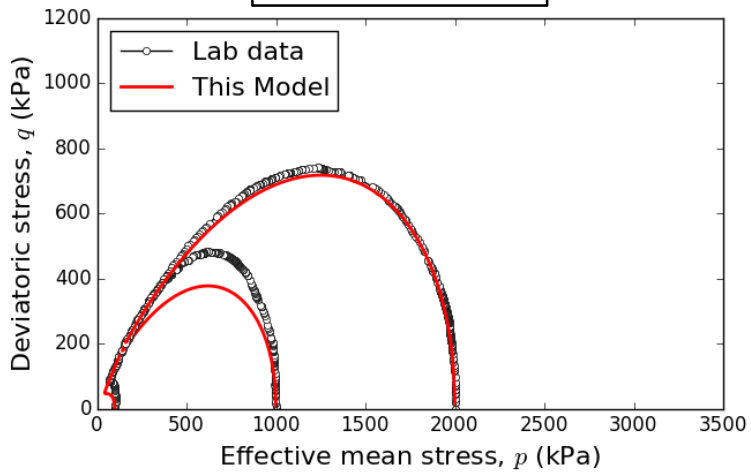


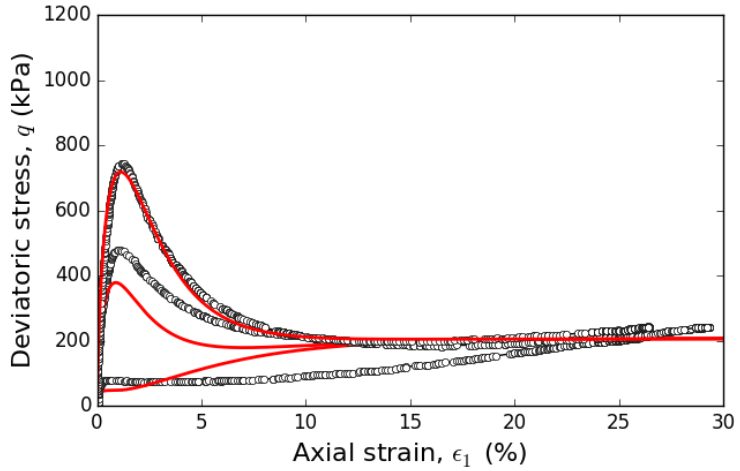
Dr = 0.379





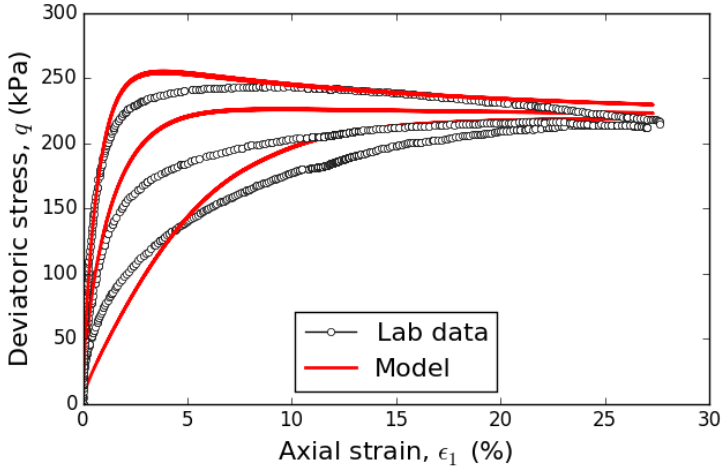
Dr = 0.185

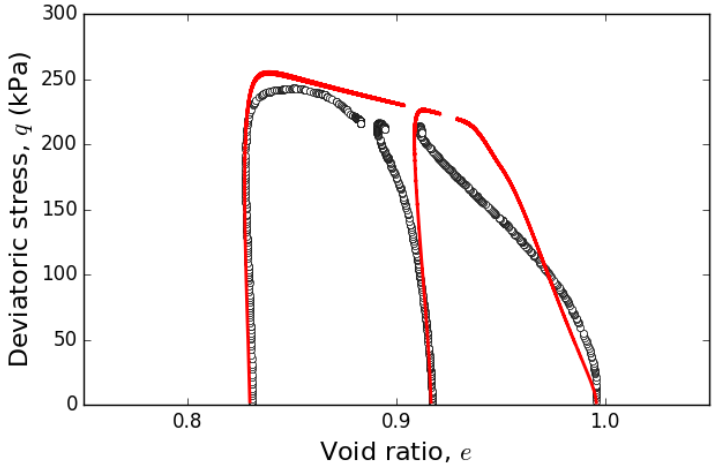




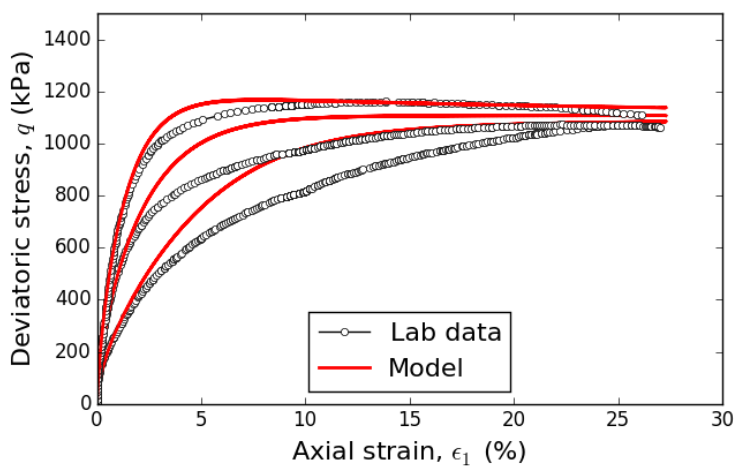
Toyoura sand
Monotonic drained triaxial simulation

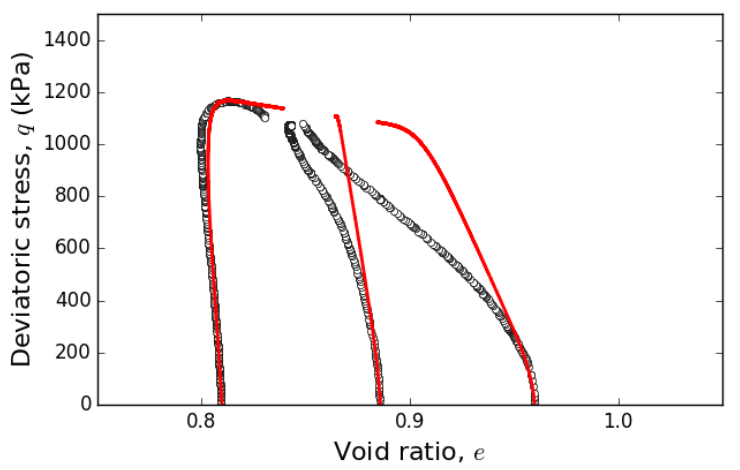
p = 100 kPa



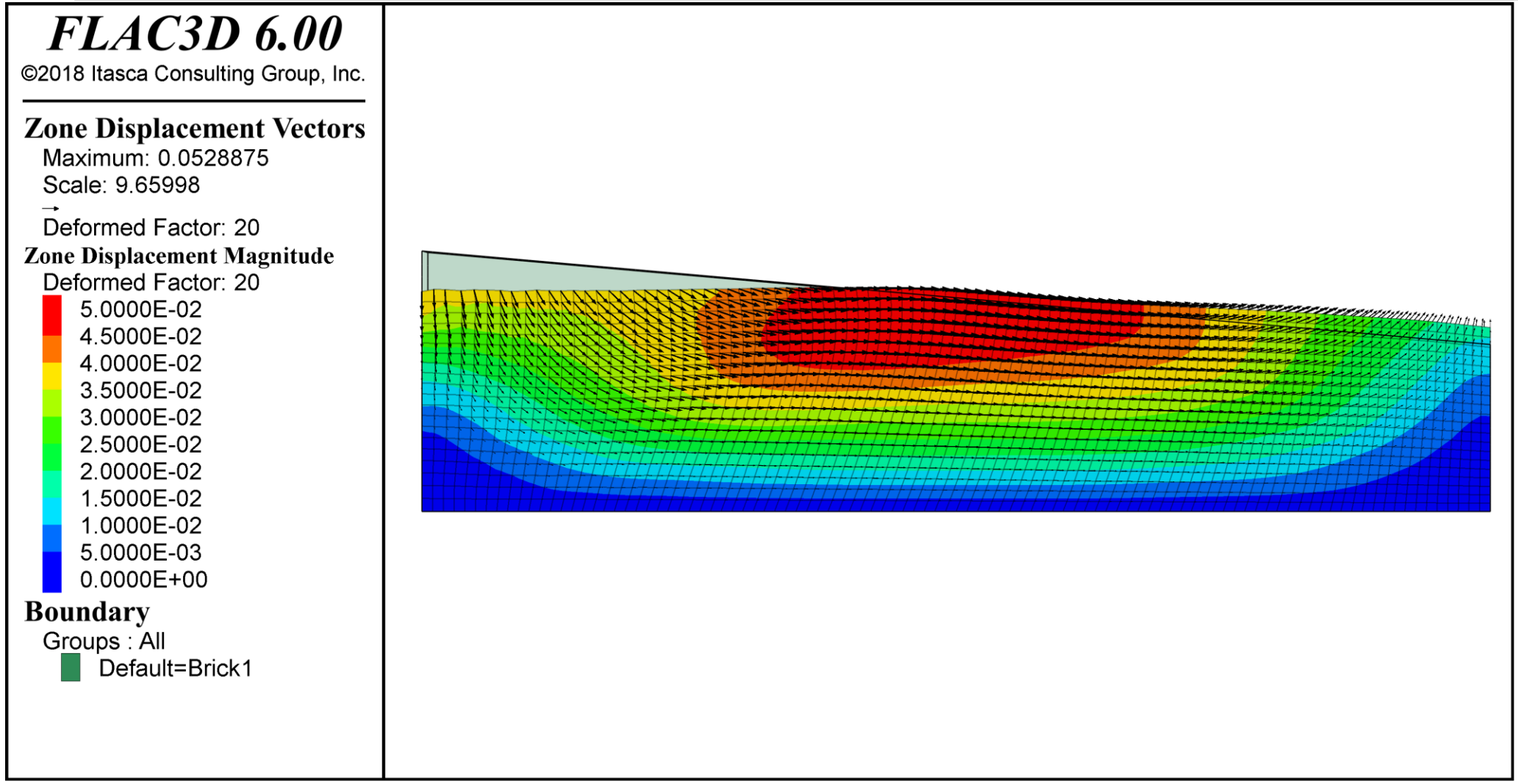


p = 500 kPa

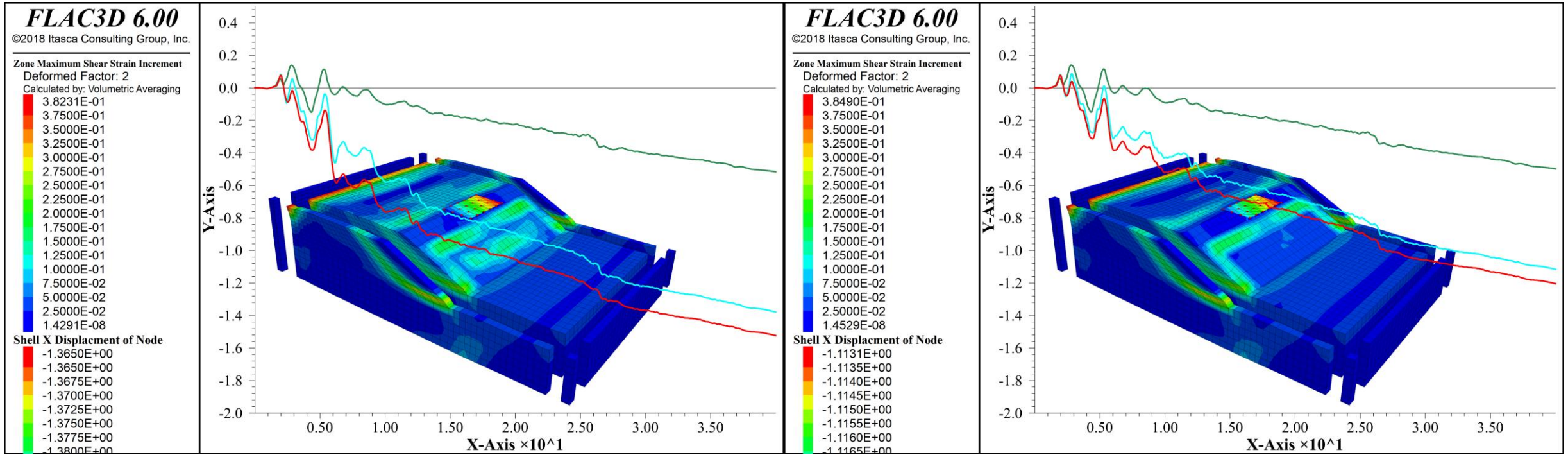




Centrifuge simulation



Wharf simulation



Dr = 0.50

Dr = 0.65

Conclusions

	UBCSAND (Version 904aR)	PM4SAND (Version 3)	PM04 (Version 2004)	This Model (Version 1.0)
Practice-friendly ?	Yes	Yes	No	Yes
CRR-$(N_1)_{60}$ curve	Match semi-empirical	Match semi-empirical	Not matching	Match semi-empirical
CSR-N curves	Ok	OK, maybe in the gentle side	Overly steep	Ok
Damping at large strain	Overly large damping	Ok	Overly large damping	Ok
Overlapped loop problem	Yes	No	Yes	No
Lode angle effect	Same as MC model	No	Yes, but not convex	Yes, always convex
Static ? Monotonic ?	Depends	Not for static; Need different calibration	OK	OK
Various densities	Need different calibration	Need different calibration	One set of parameters	One set of parameters
Kσ effect	Empirical match	Empirical match	Not matching	Intrinsically match
Kα effect (*not suggested for design*)	Trend OK	Trend OK	Not matching	Not satisfying for dense sands
Formula difficulty Formula documentation	Relatively simple More details required	Overly complex & lengthy Well documented	Relatively simple Well documented	Relatively simple Well documented
General 3D model?	No, only for plain-strain	No, only for plain-strain	Yes	Yes